

## Modeling Queuing System in Healthcare Centers. A Case Study of The Dental Department of The Kwame Nkrumah University of Science and Technology

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### Abstract

This paper analyses the use of the queuing model in healthcare, emphasizing the Outpatient and Dental Department (ODD) of a city hospital in Kumasi, Ghana. Mathematical and machine learning approaches are used to analyze patient flow, waiting, and service times. It is widely recognized that, due to high demand and limited resources in these hospitals, a hospital should promptly treat its patients, especially those in need of critical dental care. Still, surprisingly, this is not achieved in practice, particularly in government-owned health institutions, because of high demand and limited resources in these hospitals. To enhance the level of admittance to care in ODD, the hospitals should obtain a central tray setup system for instruments required for the different dental procedures done in the dental department to reduce the time spent sorting them up, thereby reducing the amount of time spent in queue and also increasing the number of dentists from one to three.

**Keywords:** Queuing theory, healthcare, ODD, m/m/s model, dental. Simulation, FCFS, patient satisfaction, arrival time, service time, Kendall's model

### 1. Introduction

Queuing theory deals with the study of queues, which abound in practical situations and arise so long as the arrival rate of any system is faster than the system can handle. Following Nkeiruka et al. (2013), the queuing theory applies to any situation in general life, ranging from customers arriving at a bank to customers at a supermarket waiting to be attended to by a cashier and in health care settings. Queuing theory is the mathematical approach to analyzing waiting lines in any setting where the arrival rate of subjects is faster than the system can handle. In queuing theory, a model is constructed to predict queue lengths and waiting times (Sundarapandian, 2009). Application of queuing theory for analysis and modeling of processes that involves waiting lines is used widely in industries for optimizing the supply of fixed resources at variable demand conditions, however, the healthcare industry views itself differently from other industry (Kritchanchai, 2012). In situations where facilities are limited and cannot satisfy the demand, bottlenecks manifest as queues but patients are not interested in waiting in queues. When patients are not interested in waiting in queues, there is a danger that waiting time will become excessive, leading to the loss of some patients to competitors.

The issue of queuing has been a subject of scientific debate for there is no known society that is not confronted with the problem of queuing. Queuing situations arise in all aspects of work

and life and are typified by the process of queuing for services, i.e., a set of physical units (people or things) which wait in a queue or queues subject to certain rules of behavior before some services are performed on or for each unit in the queue one after the other (Burodo, Suleiman and Shaba, 2019). Wherever there is competition for limited resource queuing is likely to occur Koko, Burodo & Suleiman, 2018). Queues emerge when individuals requesting service, usually called customers, arrive at a service facility and cannot be served on time (Suleiman, Burodo & Ahmed, 2022).

Customer satisfaction has become a serious concern in the service sector. Several initiatives have been introduced in the healthcare industry to enhance customer satisfaction. The healthcare industry providers globally are experiencing increasing pressure to concurrently reduce costs and improve the access and quality of care they deliver. Many healthcare institutions face long waiting times, delays, and patient queues. Long waiting times in any hospital indicate poor quality and need improvement. Due to poor hospital service operations, patients tend to spend a lot of time getting the services they need (Dellaert et al., 2015; Lohlun et al., 2015). Long queues are an indication of lack of coordination, poor management, and insufficient resources, which affects the quality of services in hospital operations and reduce patients' satisfaction (Basta et al., 2016). Managing waiting lines creates a great dilemma for managers seeking to improve the return on investment of their operations. Customers also dislike waiting for a long time. Nowadays, queueing theory can be seen as a branch of Birth-death Processes, which is, in turn, a branch of stochastic processes. Customer satisfaction has been observed as a key strategy of every business and a benchmark against which many organizations have set their standards (Burodo, Suleiman and Yusuf, 2021). Delays result from variations between a service's demand and the accessible resources to meet the demand. Queueing theory can assess customer arrival rate, service rate, customer waiting time, and staff productivity. The conventional queueing system, like long queues, can lead to unfair, frustrating, and unexplained waiting times, affecting the customer's overall satisfaction with service. Queues form when individuals requesting service, typically called patients, reach a service facility and cannot be served promptly. In healthcare delivery systems, patients are typically the customers, and either outpatient clinics or hospitals are the service facilities. A common feature of most queueing models is that customers are discrete, and the number of customers waiting in the service facilities is integer-valued. Institutions that care for sick and ill people vary enormously in scope and size, ranging from small outpatient clinics to large city hospitals. The study is designed to help the management of KNUST Hospital with employee adequacy and also help to reduce patient-waiting time for services at the dental department. However, as diets change to include sugarier snacks and fewer fruits and vegetables due mainly to globalization and a wider variety of goods, there is an increasing need for people to have regular dental check-ups and sometimes even actual dental treatment. Unfortunately, like most developing countries, Ghana lacks many dentists. Only a few dentists in Ghana can meet the needs of the Ghanaian population, especially in rural areas.

A dental clinic is an integral part of a social and medical organization that provides dental care to the population and whose outpatient services reach the family and its home environment. Nowadays, there are a lot of dental clinics in the Ashanti Region Kumasi. This is because many dentists in government hospitals in Kumasi have opened private dental clinics, reducing pressure on hospitals. Some services include pain relief (tooth extraction, temporary incision, and drainage) and dental restoration (simple amalgam filling, temporary dressing). The OPD

of a hospital act as a bridge between the hospital and the community. Hence, it is very important to plan the OPD to maximize utilization and quick turnover. (McQuarrie, 1983). It is imperative to have effective cooperation between the medical services and the support line services catering to the OPD requirements (Kritchanchai, 2012). The interpersonal skills of the medical personnel, availability of medicine, hospital infrastructure, and medical information play an important role in managing OPD and positively influencing patient satisfaction (Natarajan, 2006). Hence, it is necessary to focus on optimizing waiting time in hospital operations for the benefit and well-being of patients.

## 1.2 Problem Statement

The KNUST Hospital is one of the Kumasi-Ashanti region's hospitals with a dental department. The hospital's dental department was started in 1991. The dental department offers a full range of dental health services, including all types of fillings (amalgams, composite, and Root canal treatment), extractions, orthodontic treatment, and prosthetic treatment. The dental department seeks to:

- i. Provide quality healthcare
- ii. Increase access to dental service
- iii. To improve upon its dental service delivery
- iv. To reduce patients' waiting time.

The challenge at the dental department is that the queue is long and service is very slow.

## 2. Related Works

Queuing is a challenge for all healthcare systems. In the present world, considerable research has been done on improving queuing systems in various hospital settings. This, unfortunately, has not been the case in developing countries. This paper seeks to contribute to this subject by analyzing the queuing situation in government hospitals and bringing its practical value to enhancing decision-making in hospitals. Ok et al. (2005) indicated that patient satisfaction in a dental clinic is concerned with meeting clients' perceived needs and concerns. The dental patients' needs and concerns not only include considerations for the technical quality of the service, convenience of the service, friendly atmosphere, and cleanliness of the environment and equipment but also include respect for the time patients spend in the clinic if it is to retain its clients as well as attract others.

Gorunescu et al. (2002) designed a queuing model for bed-occupancy management and hospital planning. The paper aims to present patients' movements through a hospital department by using classical queuing theory and, on the other hand, presents a way of optimizing hospital resources to improve hospital care. The methodology developed enabled the estimation of the main characteristics of access to service for patients and hospital managers: the probability of lost demands and the mean number of patients in the hospital department (bed occupancy). The paper determined optimal bed numbers for a hospital system where the writers describe patient arrivals by a Poisson process, hospital beds as the servers, and length of stay, which were modeled using phase-type distributions.

Ajay et al. (2013), on the other hand, stressed the usefulness of queuing systems in society and that their capacity can have an important effect on the quality of human life and productivity of the process. They described the scheduling discipline and queuing networks. Their analysis

provides fundamental information for successfully designing systems that achieve an appropriate balance between the cost of providing a service and the cost associated with waiting for that service.

Dhar and Rahman (2013) worked on a case study for the Bank ATM Model. The authors used queuing theory to study the waiting lines in Brac Bank ATMs in Bangladesh. Little's theorem and the single server queuing model were used to derive the arrival rate, service rate, utilization rate, and waiting time in the queue.

They concluded that the rate at which customers arrive in the queue system is one customer per minute, and the probability of buffer overflow is the probability that customers will run away because they may be impatient to wait in the queue.

Ndukwe et al. (2011) used queuing models to work on reducing queues in a Nigerian hospital pharmacy. The work aimed to characterize the queue, describe the queue discipline of the outpatient pharmacy, institute a cross-sectional intervention by streamlining queue behaviour, and measure the impact of streamlining queue characteristics and queue discipline on patients' waiting time. When the intervention involved staff re-orientation, the streamlined process reduced waiting time. Queue discipline was strictly instituted by designed tally cards that were serially numbered. The authors concluded that hospital pharmacists should intensify efforts to reduce patient queues and improve the efficiency of services.

Kembe et al. (2012) used queuing theory to study waiting and service costs of a multi-server queuing model in a specialist hospital. The waiting and service costs were determined to determine the optimal service level. Data were collected through observations, interviews, and questionnaires. TORA optimization software was used to analyze the results. The authors indicated from the analysis that the average queue length, waiting time of patients, and over-utilization of doctors could be reduced when the service capacity level of doctors at the clinic is increased.

## **2.1 Attitude of Queuing System**

Hall et al. (2006) classified customers in the queuing system as patient or impatient. If a customer joins a queue if it exists and waits until they enter the service station to get service, they are called patient customers. On the other hand, the queuing systems enjoy customer behavior in the form of defections from the queue. There may be jockeying among the many queues. That is, a customer may switch to queues that move fast. Reneging is also possible in the queue. Here, a customer stands in the queue for some time and then leaves the system because it works slowly. The probability that a patient reneges usually increases with the queue length and the patient's estimate of how long he must wait to be served. In systems where demand exceeds server capacity, reneging is the only way to attain a 'state of dysfunctional equilibrium'. On the other hand, some customers may decide not to join the queue for some reason and may decide to return for the service later. This situation is known as balking.

Blocking occurs when a queuing system limits queue length. For example, an outpatient clinic may turn away walk-in patients when its waiting room is full. In a hospital where in-patients can wait only in a bed, the limited number of beds may prevent a unit from accepting patients.

McManus et al. (2004) presented a medical-surgical Intensive Care Unit where critically ill patients cannot be put in a queue and must be turned away when the facility is fully occupied.

This is a special case where the queue length cannot be greater than zero, called a pure loss model.

Koizumi et al. (2005) found that blocking a chain of extended care, residential, and assisted housing facilities results in upstream facilities holding patients longer than necessary. They analyzed the effect of the capacity in downstream facilities on the queue lengths and waiting times of patients waiting to enter upstream facilities. Bottlenecks could cause system-wide congestion at only one downstream facility.

As Fink and Gillett (2006) analyzed, the cost of a dissatisfied customer is not negligible; they described Waiting in line as a primary source of dissatisfaction. They mentioned that with a well-known queuing theory and integrating theory behind the Taguchi Loss Function, a manager can derive the costs associated with this dissatisfaction and that customer dissatisfaction is not just an issue at the upper specification limit but rather for each moment in time beyond the targeted wait time. They illustrated by using the Taguchi Function; it can then be seen that these costs increase beyond the upper specification limit. However, by assessing these costs and taking measures to reduce the actual or perceived waiting times, organizations can quantitatively determine the cost-benefit relationship of improved waiting lines.

Chin (2007) investigated the submittal review/approval process and used queuing theory to determine the major causes of long lead times. In his study, he explored the underlying causes of waiting in a process flow and found the improvement methods from the queuing perspective.

Georgiveskiy et al. (2002) examined the widespread context problem of extended waiting times for health services in the context of the Emergency Department (ED) at a regional hospital. The authors also examined the operations.

Research (OR) to reduce the waiting time in the hospital-admitting department. They conducted their study in five phases, from data collection to actual improvement in the quality of the health care delivery system.

### **3. Methodology**

The data used for this study were obtained from the KNUST Hospital. The arrival time, the time a patient was called to see the dentist, and the time service ended for patients were recorded. The data were gathered for two weeks (13th-24th August 2024). The queuing discipline used for this study was the first-come-first-served discipline. Sources of reading material were the Internet and the K.N.U.S.T library. The method of solution is manual. The queuing model used was the single-server queuing model. Statistical tools including R, Excel and Python were utilized for a proper analysis of our findings.

#### **3.1 Data Collection Procedure**

The researcher was taken through all the processes at the dental section. Data were collected over two weeks from 8:30 am to 11:30 am each day. The research goal during the collection of data was to get time between patient consecutive arrivals and service length. More interest was taken in periods where each of the following events occurred: patient arrival, beginning of service and end of service in the dental department. Patients coming for dental treatment are not made to join the queue at the out-patient department. They were quickly attended to and made to join the queue at the dental section. The data of most fundamental importance were

arrival time, service time and departure time of patients. These data were collected within a three-hour interval.

### 3.2 Population and Sample Size

We considered a city hospital in Ashanti-Kumasi, Ghana, and studied the admissions through its Outpatients and Dental Department (ODD). We considered all registered patients for two-week period of data collection with a sample size of 26 registered patients.

### 3.3 Queuing Model and Kendall's Notation

The basic queuing model is shown below. It can be used to model, for example, machines or operators processing orders or communication equipment processing information. Among others, a queuing model is characterized by:

**3.3.1 The customer arrival process:** Usually, we assume that the inter-arrival times are independent and have a common distribution. In many practical situations, customers arrive according to a Poisson stream (exponential inter-arrival times). Customers may arrive one by one or in batches. An example of batch arrivals is the customs office at the border, where travel documents of bus passengers have to be checked.

**3.3.2 Customer behaviour:** Customers may be patient and willing to wait (for a long time) or be impatient and leave after a while. For example, in call centres, customers will hang up when they have to wait too long before an operator is available, and they may try again after a while.

**3.3.3 The service times:** We usually assume that the service times are independent, identically distributed, and independent of the interarrival times. For example, the service times can be deterministic or exponentially distributed. It is also possible that service times are dependent on the queue length. For example, the processing rates of the machines in a production system can be increased once the number of jobs waiting to be processed becomes too large.

**3.3.4 The service discipline:** Customers can be served one by one or in batches. We have many possibilities for the order in which they enter service, including first come, first served, i.e. in order of arrival, random order, last come, first served (e.g. in a computer stack or a shunt buffer in a production line); iv. priorities (e.g. rush orders first, shortest processing time first); processor sharing (in computers that equally divide their processing power over all jobs in the system), The service capacity: There may be a single server or a group of servers helping the customers

The determination of good buffer sizes is an important issue in the design of these networks. Kendall introduced a shorthand notation to characterize a range of these queueing models. It is a three-part code  $a/b/c$ . The first letter specifies the interarrival time, the second the service time distribution, and the third signifies the number of servers. For example,  $a/b/1$  is a queuing model with a single server. In the basic model, customers arrive individually and are always allowed to enter the system. There is always room, no priority rules, and customers are served in order of arrival. It will be explicitly indicated (e.g. by additional letters) when one of these assumptions does not hold.

**Table 1: Presentation of Data Collection from 13<sup>th</sup> to 17<sup>th</sup>**

Summary of Data Collected			
Monday			
No. of patients	Arrival Time	Service time (minutes)	Time spent in queue(minutes)
1	8:38	0:06	135
2	8:49	0:19	126
3	9:02	0:18	116
4	9:31	0:24	90
5	9:32	0:12	108
6	9:47	0:05	95
Tuesday			
No. of Patients	Arrival Time	Service time (minutes)	Time spent in queue(minutes)
1	8:32	0:20	118
2	8:36	0:19	119
3	8:40	0:04	120
4	9:00	0:18	119
5	9:10	0:20	135
Wednesday			
No. of customers	Arrival Time	Service Time	Time spent in queue(minutes)
1	8:40	0:14	140
2	8:52	0:16	133
3	9:39	0:23	94
4	10:14	0:05	45
5	10:17	0:05	64
Thursday			
No. of customers	Arrival Time	Service Time	Time spent in queue(minutes)
1	8:39	0:15	92
2	8:48	0:14	85
3	8:55	0:06	73
4	10:15	0:04	11

Table 1 shows a sample of the data collected on the number of patients who arrived at the dental department from Monday to Friday during the data collection. The first column shows the patient number, while the second and third columns show the arrival and service times, respectively. The fourth column shows the time spent in the queue for each patient.

#### 4.5 Analysis of Data Collected

The data collected from the dental department at Kwame Nkrumah University of Science and Technology Hospital through self-recording was integrated into Microsoft Excel for a better data visualization. Utilizing Python programming packages including pandas and numpy, we estimated the mean arrival rates and mean service rates, and their results are used to measure the performance of the entire system.

##### 4.5.1 Queuing Model

##### 4.5.2 Kendall's Notation

According to the Kendall notation, a queue is therefore described in a shorthand notation by  $A/B/C/K/N/D$  or the more concise  $A/B/C$ . In the concise version, it is assumed that  $k=\infty$ ,  $N=\infty$  and  $D=FCFS$  (first come, first serve)

**Table 2: Description of Kendall's notation**

Letter	Description
A	The arrival process
B	The service time distribution
C	The number of servers
K	The number of channels in the system
N	The calling population
D	The queue discipline

### 4.5.3 Exponential Queuing Model

The research assumed an infinite population; the arrival and service rates are assumed to be independent, with a first-come-first-served discipline.

$\lambda$  = arrival rate

$s$  = number of servers

Utilization factor:  $p = \frac{\lambda}{\mu}$

The average number of patients in the system:  $L_s = W_q + \frac{1}{u}$

The average number of patients in the queue:  $L_q = \frac{p^2}{1-p}$

The average time spent in the queue:  $W_q = \frac{p}{u-\lambda}$

The average time spent in the system:  $W_s = \frac{1}{u-\lambda}$

### 4.5.4 THE M/M/S MODEL

The most commonly used queuing model is the M/M/s or Erlang delay model. This model uses a single queue with unlimited waiting rooms that feed into identical servers. The selected queuing model suggests that daily admission rates follow a Poisson distribution and that the service times are distributed exponentially, and since ODD generally runs independently of other services, its capacity needs can be assessed regardless of other parts of the hospital. We can take the current operating characteristics of a given ODD as arrival rate, service rate, and number of servers. (These two assumptions are often called Markovian; hence, the two are used in the notation used for the model). One advantage of using the M/M/s model is that it requires only three parameters to obtain performance estimates with very little data. Given an average arrival rate  $\lambda$ , average service duration  $\mu$ , and the number of servers, easy-to-compute formulae are available to obtain performance measures such as the probability that an arrival will experience a positive delay.



$$P_0 = \left[ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c! (1 - \frac{\rho}{c})} \right]^{-1}$$

$$U = \sum_{i=1}^N \frac{\lambda_i}{c\mu_i}$$

$$P_o = \left[ \sum_{i=0}^m \left( \frac{M!}{(M-1)!} \left( \frac{\lambda}{\mu} \right)^i \right) \right]^{-1}$$

$$P_n = \left[ P_o \left( \frac{\lambda}{\mu} \right)^n \frac{M!}{(M-n)!} \right], 0 < n \leq M, \text{ or } n > M$$

**4.5.5 Results of characteristics of our system**

**TABLE 3: Arrival and Service rates for the week.**

Day	Mean arrival rate (patients/hour)	Day	Mean service rate (patients/hour)
Monday	2	Monday	4.2857
Tuesday	1.6667	Tuesday	3.7037
Wednesday	1.6667	Wednesday	4.7619
Thursday	1.3333	Thursday	6.1538
Friday	1.3333	Friday	3.3333

Table 2 shows the mean arrival rate and service rates for the week. The first column shows the days, while the second column shows the mean service rates for the entire week.

**TABLE 4: Result for Utilization factor for the week.**

Day	Utilization factor (patients/hour)
Monday	0.4667
Tuesday	0.4501
Wednesday	0.3505
Thursday	0.2167
Friday	0.4112

The table above shows that the server's busiest day was Monday, with a utilization factor of 46.67%, followed by Tuesday, with a utilization factor of 45%. The lowest utilization factor was 21.67%, and it occurred on Thursday.

**TABLE 5: Result for Operating Characteristics for the week**

Operating Characteristics	DAYS				
	Monday	Tuesday	Wednesday	Thursday	Friday
$\mu$ (patients/hour)	4.2857	3.7037	4.7619	6.1538	3.3333
$\lambda$ (patients/hour)	2.0013	1.6667	1.6667	1.3333	1.3333
$L_s$ (patients)	0.8750	0.8182	0.5385	0.2766	0.6667
$L_q$ (patients)	0.4084	0.3682	0.1885	0.0599	0.2667
$W_s$ (hours)	0.4375	0.4909	0.3231	0.2074	0.5002
$W_q$ (hours)	0.2042	0.2209	0.1131	0.449	0.2121
$\rho$	0.4667	0.4523	0.3541	0.2167	0.4023

The analysis in Table 3 shows that Monday recorded the highest number of patients (six patients), with Thursday and Friday recording the lowest of four patients. The arrival rate of patients on Monday was the highest, with an arrival rate of 2 patients/hour. The lowest arrival rate was estimated at 1.3333 patients/hour and occurred on Thursday and Friday. The highest service rate was also found to be 6.1538 patients/hour, which occurred on Thursday; the lowest service rate, estimated to be 3.3333 patients/hour, occurred on Friday. The least of the average number of patients in the system was estimated to be 0.2766 patients and occurred on Thursday, and this corresponded to the least average time spent by a patient from arrival until fully served, which is 0.2074 hours. Also, the largest of the average number of patients in the system was estimated to be 0.875 and recorded on Monday. The highest average time a patient spent from arrival until fully served was 0.4909 hours recorded on Tuesday.

#### 4.6 Comparison of Operating Characteristics for different numbers of servers using average arrival and service rates.

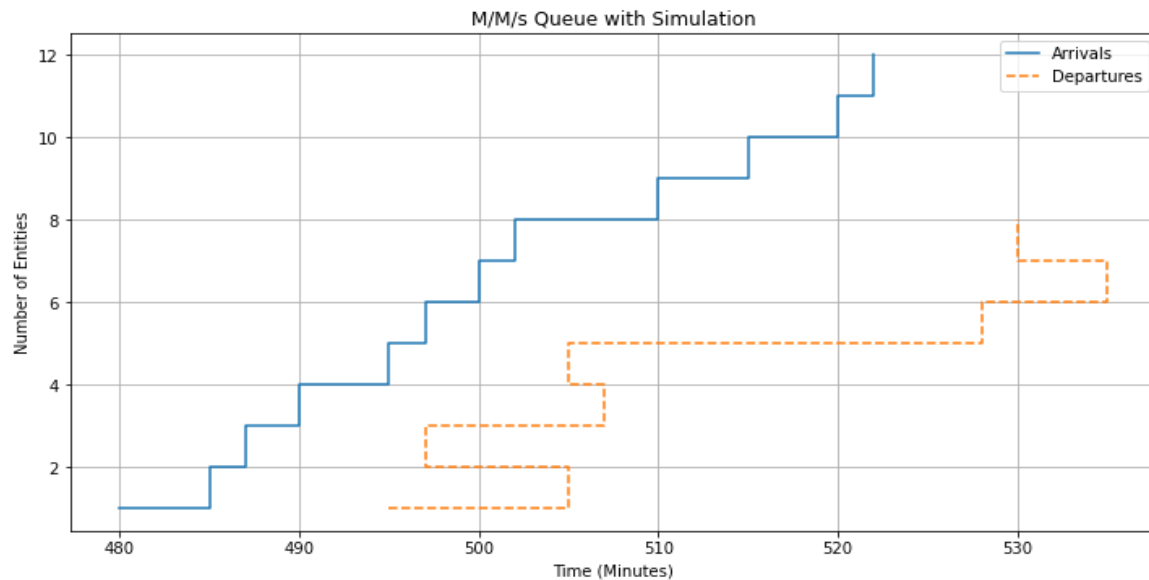
The average arrival and service rates at the dental department are computed by finding the sum of the arrival rates and service rates and dividing by five. The average arrival rate during the five-day data collection period is represented in the table below.

**Table 6: Summary of the Operating Characteristics for different number of servers.**

Operating Characteristics for Different Numbers of Servers	Queue Model		
	m/m/1	m/m/2	m/m/3
$L_s$	0.5619	0.3718	0.3606
$L_q$	0.2021	0.0121	0.0008
$W_s$	0.3512	0.2324	0.2254
$W_q$	0.1263	0.0075	0.0005
$\rho$	0.3597	0.1799	0.1199

#### 4.7 Application of Machine Learning to Simulate Patient Flow

This research has attempted to apply machine learning algorithms to simulate the patient flow behavior at the Dental Department of Kwame Nkrumah University of Science and Technology Hospital and offered ample information to hospitals interested in using queuing theory to enhance the quality of healthcare services. In the proposed model, we concentrate on queuing system management, patient waiting time, and the arrival time and service rate of each server.



**Fig 1: Simulation of Queue Behaviour.**

The figure above shows the behavior of the queuing for the week at the dental department of Kwame Nkrumah University of Science and Technology Hospital. The queue at the Outpatient department grows longer as fewer patients are served in the shortest possible time.

#### 4.6 Discussion and Results

From Table 3, the busiest of all the days was Monday since it recorded the highest number of patients, and the percentage of time the server is being utilized by a patient (server utilization) was 46.67%. This was followed by Tuesday. This may be because the dental department is closed on weekends, so people with dental issues over the weekend must wait until Monday. Thursday recorded the least of 21.67%. The table also shows that more patients wait in queue on Mondays than any other day. The average time a patient spends from arrival until fully served on Monday is 0.4375 hours, which is the highest compared to all the other days. On Thursday, the average time a patient spends until fully served is 0.2074 hours, which is the lowest compared to all the other days. Also, Thursday recorded the lowest average time it takes a patient to start being served, while Monday recorded the highest, followed by Tuesday, Friday, and Wednesday. An extension of the working days from Monday to Saturday could further improve the utilization factor on Monday.

From Table 5, it is observed that there is a reduction in the value of the operating characteristics as the number of servers increases. An increase in the number of servers from one to two indicates that the servers will be 17.99% busy at the dental department. The average number of patients in the system reduced from 0.5619 patients to 0.3718 patients. The average number of patients in the queue decreased from 0.2021 patients to 0.012 patients. The average time a

patient spends from arrival until fully served decreased from 0.3512 hours to 0.2324 hours, and the average time it takes a patient to start being served reduced from 0.1263 hours to 0.0075 hours.

Finally, increasing the number of servers from two to three also shows a decrease in the operating characteristics. The server utilization factor reduced from 17.99% to 11.99%. The average number of patients in the system reduced from 0.3718 patients to 0.3606 patients. The average number of patients in the queue decreased from 0.012 patients to 0.0008 patients. Also, the average time a patient spends from arrival until full service was reduced from 0.2324 hours to 0.2254 hours, and the average time it takes a patient to start being served was reduced from 0.0075 hours to 0.0005 hours. Increase the number of servers at the dental department can be daunting as the spaces of most dental care rooms are not enough to accommodate more server. However, creating a separate housing to accommodate more servers for dental treatment could optimize the waiting time and service rate at the Dental Department.

## **5. Conclusion**

Patients' satisfaction is very important to hospital management because patients are the people who sell the hospital's good image to others, which helps to increase the hospital's revenue. The objective of every health facility is to reduce patients' waiting time, increase revenue, and improve customer service and care.

The study examined the queuing system at the dental department of Kwame Nkrumah University of Science and Technology Hospital, examining patients' arrival rates, service rates, and utilization factors in the system. These three parameters were then used to measure patients' waiting time in the queues and the entire system and to find the number of patients in the queue and the whole system.

The analysis of the study showed that Monday is the busiest day at the dental department because it recorded the highest number of patients (6 patients), with Thursday and Friday recording only four patients. This may be because the dental department is closed on weekends, so people with dental issues over the weekend must wait until Monday. During the data collection period, it was observed that most of the patients who spent longer time in the dental treatment room had major dental issues like tooth extraction. Patients with minor issues like gum bleeding spent less time in the dental treatment room. The analysis revealed that the operating characteristics of the single-server model were very high compared to those of the two-server and three-server models indicating that the higher the service quality, the better satisfaction patients derived from the services.

### **5.1 Future Advancements**

There are various ways for future research. A challenging and interesting step is to extend our approach to the case of many customer types with different mean service and patience times. It is also interesting to consider general service-time distributions. Another useful extension would be to consider protocols with two priorities, ie pre-emptive resume rule, where patients with severe dental problem can interrupt the service of patients with normal dental routine. This simply means that a server currently serving a normal dental patient, while a new severe dental patient enters the system, will stop and serve severe dental patient before turning to the normal dental patient.

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