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Application of Queuing Theory and Management of Waiting Time Using Multiple Server Model: Empirical Evidence From Ahmadu Bello University Teaching Hospital, Zaria, Kaduna State, Nigeria

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Abstract

In many hospitals, time spent by the patients to get service in the healthcare facility is always the main problem faced by the health care providers. This trend is on the increase and it is a potential threat to healthcare services. In Nigeria, general hospitals with large number of patients have cases where patients may not be attended to on time. The consequences of keeping patients in a queue for too long in order to get medical care can cause a slew of problems or even lead to death. In this study, the multiple servers queuing models were used to analyze service efficiency of the Ahamadu Bello University Teaching Hospital Zaria, Kaduna State. Primary data was collected through observation and questionnaire methods at the hospital over a two-week period to determine the queuing model that minimizes patients waiting time. The results showed that a significant percentage of the patients were not satisfied with how queues management strategies put in place by the hospital. Regression analysis shows that all the dimensions of service quality have significant positive relationship with the patient's satisfaction.

Keywords: Queuing theory, Queuing management, Service Quality, Patients satisfaction, m/m/i model, m/m/s model, arrival time, service time, and utilization factor.

1. Introduction

Around 1909, a Danish engineer named Erlang developed queueing theory as a result of his research work (Winsotn, 1991). He conducted an experiment and discovered that telephone traffic demand fluctuates. Following that, he released a report on the delays in automatic dialing equipment. Shortly after his published work and during the end of World War II. Erlang's early work was extended to more general challenges and business applications of waiting lines and to various service industries. Modeling a service industry as a queuing system has a number of benefits, including diagnosing problems and identifying restrictions in order to better understand real-world systems rather than making specific predictions about them. Queuing theory is the mathematical study of waiting lines, or the act of joining a line (queues). In queuing theory a model is built so that queue lengths and waiting times can be calculated (Sundarapandian, 2009). The issue of queuing has been a subject of scientific debate for there is no known society that is not confronted with the problem of queuing. Queuing situations arise in all aspects of work and life and are typified by the process of queuing for services, i.e., a set of physical units (people or things) which wait in a queue or queues subject to certain rules of behavior before some services are performed on or for each unit in the queue one after the other (Burodo, Suleiman and Shaba, 2019). Wherever there is competition for limited resource queuing is likely to occur Koko, Burodo & Suleiman, 2018). Queues emerge when individuals requesting service, usually called customers, arrive at a service facility and cannot be served on time (Suleiman, Burodo & Ahmed, 2022).

Waiting time is define as the time spent by the patient in the queue before the commencement of the services (Shanmugasundaram & Umarani, 2015). Unfortunately, this case happen in many teaching hospitals. Teaching hospitals received a large number of patients every day and this generally results in long patient waiting time (Afrane & Appah, 2014). Waiting for service is a common phenomenon in all setting where patients are involved, such as hospitals, gas stations, department stores, restaurants and banks (Adewole, 2016). Whenever patients arrive at a service facility, some of them have to wait before they receive the desired service. It means that the patients has to wait for his/her turn. This situation can be frustrating when patients have to wait for a long time before they are served in a queue, this leads to patients dissatisfaction as they cannot have the desired level of service (Nkrumah Yeboah & Adiwokor, 2015). Similarly, some of the patients die waiting for their turn of service. Service is delayed when the service demand is more than the available capacity. When there is low capacity in comparison to the demand then the queue will form in the system (Ahmed & Ali, 2021).
Queue management is a critical part of the service industry. It deals with the issue of fair treatment of customers in order to reduce waiting time and improve service quality (Desta and Belele 2019). Queue management deals with cases where the customer arrival is random; therefore, service rendered to them is also random. According to Lee (2019) queue management refers to set of principles aimed at customer flow and streamlining the queuing experience. Unmanaged queues are detrimental to the gainful operation of service systems and results in a lot of other managerial hitches (Chase, Aquilano, and Jacobs, 2001).

Customer satisfaction has been observed as a key strategy of every business and a benchmark against which many organizations have set their standards (Burodo, Suleiman and Yusuf, 2021).

In order to contribute to the ongoing argument, this paper analyzed queuing theory and management of waiting time in hospitals using multiple servers: empirical evidence from Ahmadu Bello University Teaching Hospital, Zaria, and Kaduna State, Nigeria. To achieve this objective, the study is structured into five sections including introduction; section two reviews empirical literature while section three details the techniques of analysis. Section four analyzes the results and the last section concludes and details possible recommendations.

1.1 Statement of Problem

The hospital industry, like other service-oriented businesses, is becoming increasingly competitive. Providing quick service by a service provider improve patients satisfaction. The fundamental issue confronting hospitals in Nigeria is that attendance has increased as a result of increase in population and awareness among Nigerians, hospital/clinic capacity has remained constant, and patients must wait a long time for service. Furthermore, multiple studies showing patient dissatisfaction with extended waiting periods have been found in the literature, indicating that this is a widespread problem in hospital practice and a common source of worry and unhappiness among patients.

In addition, extra hands are required in order to increase the service rate, which implies cost to management. Furthermore, the obvious costs implications of patients waiting range from idle time spent when a queue forms, which results in man-hours to a loss of good will, which can occur when patients are dissatisfied with a system. This study seeks to determine amount of time a patient is likely to experience in a system at Ahmadu Bello University Teaching Hospital, Zaria.

2. Literature Review

This section examines previous scholarly empirical studies on the application of queuing theory in hospitals services within and outside Nigeria. This is to allow comparisons because majority of the case studies are similar in a number of ways. However, the few studies selected are reviewed below.

Kha skheli et al., (2020) used optimum number of receptionists and doctors to optimize the performance of existing queuing systems at the out-patient departments (OPDs). The most congested OPD i.e. medical OPD was selected for the study at the case hospital 1 and then same OPD was selected in another public sector hospital (case hospital 2) in Sindh Pakistan. The study collected data for two weeks. The data collection parameters were; arrival rate, service rate of patients, number of servers, salaries of the servers and associated waiting cost of patients. Arrival and service distribution of the patients were verified as per assumptions of the multi-server queuing model (M/M/c) by using input analyser of Rockwell Arena 14.5. Performance measures of the queuing system were calculated by using TORA optimization software. For cost calculation and graph plots MS excel was used. According to the results, one receptionist and doctor was suggested to be increased at both of the OPDs for the minimization of congestion of patients and their waiting times.

Segun (2020) conducted a research on the Performance Modelling of Health-care Service Delivery in Adekunle Ajasin University, Akungba-Akoko, Nigeria Using Queuing Theory. The purpose of this study was to determine the waiting, arrival and service times of patients at AAUA Health- setting and to model a suitable queuing system by using simulation technique to validate the model. This study was conducted at AAUA Health- Centre Akungba Akoko. It employed analytical and simulation methods to develop a suitable model. A stopwatch was used to calculate the number of minutes spent by each patient from the reception section where patients arrive and collect their hospital cards or register to the last section (the consulting room section). Data on the arrival time, waiting time and service time of each patient was collected on Weekdays (Mondays through Fridays) for three (3) weeks. The data was calculated and analyzed using Microsoft Excel. Based on the analyzed data, the queuing system of the patient current situation was modelled and simulated using the PYTHON software. The result obtained from the simulation model showed that the mean arrival rate of patients on Friday week1 was lesser than the mean service rate of patients (i.e. 5.33> 5.625 ($\lambda > \mu$). What this means is that the waiting line would be formed which would increase indefinitely; the service facility would always be busy. The analysis of the entire system of the AAUA health centre revealed that queue length increases when the system is very busy. The study recommends the need for the AAUA Health-Centre to improve the quality of service offered to the patients visiting this health centre.

Nor and Binti (2018*)* Applied Queuing Theory Model and Simulation to Patient Flow at the Outpatient Department. The objective of this study is to determine the waiting arrival time and service time of patients at the outpatient counter and to model suitable queuing system using simulation technique. This research was carried out at a Public Health Clinic in southern Malaysia. Descriptive analytical and simulation method were employed to develop suitable model. The collection of waiting time for this study is based on the arrival rate and service rate of patients at the outpatient counter. The data calculated and analyzed using Microsoft Excel. Using the ARENA, the patient’s existing queuing system was modelled and simulated based on the analysed data. Descriptive analysis and observations study was used to determine the time taken of patients from the registration until seen by pharmacist at the outpatient clinic. The results obtained from the ARENA simulation stated that the average waiting time of patient have to wait before get the treatment is 54.295 minutes whereas the maximum waiting time is 144.48 minutes. Then, the average service time for patient get the treatment is 13.48 minutes whereas the maximum service time for several patients is 23.724 minutes. Therefore, the average total time spend by patients in outpatient department is 68.315 minutes and maximum total time in system is 156.718 minutes. Total average number of patients arrived at outpatient counter is 327 patients per day. Thus, based on the result average total number of patient gives the utilization of server at outpatient department is 78.84%.

Shastrakar and Pokley (2017) analyzed different parameters of queuing theory for the waiting time of patients in hospital, in their research parameters such as arrival rate, service rate, utilization factor, the average number of patient in the system, average number of patient in the queue, average time spent by the patient in the system, average time spent by the patient in the queue were analyzed. They concluded that the percentage of idle workstation is very less and utilization factor is very high so it needs some improvement in the service facility.

Rotich (2016) conducted a study on the effect of Queuing Theory on Emergency Medical Service Department in Moi Teaching and Referral Hospital (MTRH). The objective of this paper was to determine the optimum waiting and service cost in a hospital ICU emergency service. The study adopted Use of M/M/s queuing model to analyze ICU services using secondary data of MTRH emergency patient’s arrival and service rates together with estimated

service cost of available 6 beds. Waiting cost estimated using formulated Modified Normal Loss function. Modeling and simulation method were employed to develop suitable model. The results showed that the average individual tolerance of 0.083 hours and average response time of 0.083hrs, the present scenario of 6 ICU beds in MTRH is operating at a service cost of Ksh 60 and patient queuing cost of Ksh 415.53 per hour. The length of the queue is 1.4 hour or approximately 34 patients per day. Increasing ICU beds to 18 minimizes the length of the queue to 6 patients per day and queuing cost by 76% and reduces the total cost by 65% thereby reducing the financial burden of the patients and increase the chances of saving lives during emergency cases. However, the management of the hospital need further work and inclusion of related services to give a bigger and better picture of the facility.
Even though there have been many research undertaken on the subject matter both within and outside the country, there are gaps that have been identified from past studies on the issue.

In light of the above empirical reviews, it is clearly that many researchers have conducted research on Application of Queuing Theory in hospital services in Nigeria and other part of the world. But the researcher could not find any research work that was written on Application of Queuing Theory and Management of Waiting Time in Hospitals specifically using Ahmadu Bello University Teaching Hospital, Zaria as a case study.
Moreover, in terms of methodological gap, the above empirical studies used different tools of analysis. For example, Segun (2020); Nor and Binti (2018) employed analytical and simulation methods to develop a suitable model. However, Rotich (2016) adopted modeling and simulation methods. In the same vein Khaskheli et al., (2020) multi-server queuing model (M/M/c) by using input analyser of Rockwell Arena 14.5. To fill the existing gap, this study employed primary data through observation and questionnaire methods as tools of analysis.
Similar researches were conducted in south- west zone (Segun, 2020).Thus, such research has not been conducted in North-west zone.

3. Methodology

3.1 Sources of Data Collection

In this study, primary data were collected through observation and questionnaires administration.

3.2. Population and sample size

Ahamadu Bello University Teaching Hospital Zaria was considered in this research with a population size of 4,829 registered patients. According to Raasoft sample size calculator, a minimum of 400 sample size should be considered. A sample size of 400 of registered patients was used in this research.

3.4. Sampling technique

The hospital used in this research was selected using convenience sampling technique. A sample of registered patients in this hospital was selected using simple random sampling technique.

3.5 Data Analysis

Data obtained through questionnaire were analyzed using percentages and frequencies. Similarly, exponential queuing models were developed using the data obtained through observation.

3.5. Exponential Queuing Model

Certain assumptions that will be made during the research are:

- Queue discipline will assume to be first-come-first-serve (FCFS) type.
- Reneging, balking and jockeying of the patients will not be taken into consideration in the study.
- The population source is going to be infinity.
- Infinite number of patients is allowed in the system.

According to Kendall's notation, the model for the system could be represented as m/m/i: FCFS/∞/∞. The following parameters would be analysed for the model:

Mean patients arrival rate = λ

Mean service rate = μ

Utilization factor, P = $\frac{\lambda}{\mu}$

Probability of zero patients in the system $P_o = 1 – P$

Probability of having n patients $Pn = P_oP^n = (1 – P)\ p^n$

Average number of patients in the system, Ls = $\frac{\lambda}{\mu-\lambda}$

Average number of patients in the queue, Lq = Ls - $\frac{\lambda}{\mu}$

Average time spent in the system, Ws = $\frac{1}{\mu-\lambda}$

Average time spent in the queue, Wq = $\frac{P}{\mu-\lambda}$

3.5.1. Multi-Server Model M/M/S

$$P_0 = 1\Big/\left[\sum_{n=0}^{s-1}\frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\frac{1}{1-\lambda/(s\mu)}\right]$$

$$L_q = \frac{P_0(\lambda/\mu)^s \rho}{s!\,(1-\rho)^2}$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_s = W_q + \frac{1}{\mu}$$

4. Results and Discussion

4.1 Socio-Demographic Profile of Patients

Table 1 presents the socio-demographic characteristics of 389 respondents' retrieved questionnaires out of 400 administered. It indicates that 170(43.6%) and 219(56.4%) of the respondents are male and female respectively. The age distribution of the respondents indicates that 51(13.2%), 177(45.6%), 127(32.3%) and 34(8.9%) aged less than 20 years, 21 to 40 years, 41 to 60 years and above 60 years respectively. Educational qualification profile of the patients indicate that 68(17.4%), 98(25.3%), 128(32.9%) and 95(24.5%) possessed Quranic/Islamiyya, Primary school certificate, Secondary School certificate and tertiary certificate respectively. Employment status profile of the patients revealed that 72 (18.4%), 46(11.8%), 182(46.9%), 59(15.1%) and 30(7.8%) of the respondents are civil servants, retired workers, self-employed, students and other specified employments respectively. Finally, marital status of the patients revealed that 40(10.4%), 237(60.8%), 7(1.8%), 69(17.8%) and 36(9.2%) of the respondents are single, married, separated, divorced and widowed respectively.

Table 1: Socio-Demographic Characteristics of Patients

Gender	**Frequency**	**Percentage (%)**
Male	170	43.6
Female	219	56.4
Total	389	100.0
Age	**Frequency**	**Percentage (%)**
$\leq$ 20	51	13.2
21-40	177	45.6
41-60	127	32.3
Above 60	34	8.9
Total	389	100.0
Highest educational qualification	**Frequency**	**Percentage (%)**
Quranic/Islamiyya School	68	17.4
Primary School	98	25.3
Secondary school	128	32.9
Tertiary school	95	24.5
Total	389	100.0
Employment status	**Frequency**	**Percentage (%)**

Civil servant	72	18.4
Retired	46	11.8
Self employed	182	46.9
Student	59	15.1
Other specify	30	7.8
Total	389	100.0
Marital status	**Frequency**	**Percentage (%)**
Single	40	10.4
Married	237	60.8
Separated	7	1.8
Divorced	69	17.8
Widowed	36	9.2
Total	389	100.0

Source: Field Data, 2021

4.2 Queue Management

Table 2: Queue Management

Statement	Strongly agree	Agree	Undecided	Dis Agree	Strongly Disagree
Queues are managed properly at the hospital	25(6.3)	146(37.6)	7(1.7)	170(43.8)	41(10.6)
There are barriers to guide patients in queues	70(17.9)	174(42.1)	4(0.9)	117(30.1)	24(6.2)
Hospital staffs are of great help in helping patients in queues	45(11.6)	81(20.7)	5(1.2)	181(46.7)	77(19.8)
Queue discipline follows FCFS pattern mostly (i.e the first to arrive will be serviced firstly	65(16.8)	92(23.6)	7(1.8)	155(39.7)	70(18.1)
Generally I am satisfied with how patients are handled at the hospital while waiting for service	38(9.7)	103(26.4)	2(0.6)	206(53.0)	40(10.3)

Source: Field Data, 2021

Table 2 indicates that patients have not agreed to greater extent that queues are managed properly at the hospital and also believed that there are barriers to guide patients in queues. Similarly, patients have to some extent disagreed that hospital staffs are of great help in helping patients in queues, queues discipline follows FCFS pattern mostly (i.e the first to arrive will be serviced firstly). The table also indicated that the patients are generally dissatisfied with how they are handled at the hospital while waiting for service.

4.3 Regression Analysis

Table 3 revealed that service quality dimension of assurance, empathy, reliability, responsiveness and tangible have significant relationships patients' satisfaction. This therefore indicates that the patients believed that the higher the quality of service in terms of assurance, empathy, reliability, responsiveness and tangible, the better satisfaction that they will derive from the hospital service.

Table 3: Coefficients of regression analysis

	Coefficients	**T-statistics**	**P-values**	**Decision**
Assurance	0.678	5.681	0.000	Significant
Empathy	0.441	5.681	0.000	Significant
Reliability	0.499	3.623	0.001	Significant
Responsiveness	0.104	2.268	0.026	Significant
Tangible	0.338	4.019	0.000	Significant

4.4 Exponential Queuing model

In order to build exponential model, the study considered outpatients from the Department of Obstetrics and Gynaecology Ahmadu Bello University, Teaching Hospital Zaria. Observations were made during the fourth week of the month of August 2021 for Tuesday and Friday as days reserved for consultations in the hospital. Twenty six (26) patients arrived for consultation within a period of sixty (60) minutes on Tuesday as shown in table 4. Similarly, twenty (20) patients arrived for consultation within a period of sixty (60) minutes on Friday as shown in table 5

Table 4 Number and Time of patient's arrival on Tuesday, ABUTH Zaria

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Arrival rate in minutes	2	6	7	11	13	16	23	26	28	31	33	37	38	41	46	47	49	51
Number of Patients	19	20	21	22	23	24	25	26										
Arrival rate in minutes	53	55	57	58	59	60	62	64										

Source: Field Data, 2021

Table 5: Number and Time of patient's arrival on Friday FMC ABUTH, Zaria

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Arrival rate in minutes	3	7	11	12	16	21	27	31	33	37	38	40	41	43	45	48	50	54	57	59

Source: Field Data, 2021

4.3.1 Analysis of Data Collected on Tuesday at the Consulting Rooms

a) Inter arrival rate $= (6-2)+(7-6)+\cdots+(62-60)+(64-62)$

$= 4+1+\cdots+2+2$

$= 62$ Minutes

Mean arrival rate $\lambda = \frac{62}{26} = 2.4$ minutes per Patients

Table 6: Number and Time Patients were served at the Consulting Rooms on Tuesday

Number of Patients	Service Time in Minutes			
	Consulting Room 1	Consulting Room 2	Consulting Room 3	Consulting Room 4
1	6	12	7	13
2	20	19	22	22
3	31	26	30	34
4	36	35	34	45
5	42	44	42	56
6	50	66	56	67
7	57	78	70	76
8	61	87	76	85
9	74	93	82	90
10		105	88	96
11		115	95	107
12		133	103	118
13			117	130
14			135	

Source: Field Data, 2021

Inter service rate (Consulting room 1) $= (20-6)+(31-20)+\cdots+(61-57)+(74-61)$

$= 14+11+\cdots+4+13$

$= 68$ Minutes

Mean service rate $\lambda = \frac{68}{9} = 7.6$ minutes per Patients

Inter service rate (Consulting room 2) $= (19 - 12) + (26 - 19) + \cdots + (115 - 105) + (133 - 115)$

$= 7 + 7 + \cdots + 10 + 18$

$= 121$ Minutes

Mean service rate $\lambda = \frac{121}{12} = 10.1$ minutes per Patients

Inter service rate (Consulting room 3) $= (22 - 7) + (30 - 22) + \cdots + (117 - 103) + (135 - 117)$

$= 15 + 8 + \cdots + 14 + 18$

$= 128$ Minutes

Mean service rate $\lambda = \frac{128}{14} = 9.1$ minutes per Patients

Inter service rate (Consulting room 4) $= (22 - 13) + (34 - 22) + \cdots + (118 - 107) + (130 - 118)$

$= 9 + 12 + \cdots + 11 + 12$

$= 117$Minutes

Mean service rate $\lambda = \frac{117}{13} = 9$ minutes per Patients

Mean service rate of the 3 consulting rooms= $\frac{7.6+10.1+9.19}{4} = 9.0$

Number of servers k = 4

Mean combined rate of all servers= $K\mu = 4(9.0) = 36$

Utilization factor of the entire system= $\rho = \frac{\lambda}{k\mu} = \frac{2.4}{4\times 9.0} = 0.07\ or\ 7\%$

The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left[\left\{\sum_{n=0}^{k-1} \frac{1}{n!} (\frac{\lambda}{\mu})^n\right\} + \frac{1}{k!} (\frac{\lambda}{\mu})^k \left(\frac{k\mu}{k(\mu-\lambda)}\right)\right]^{-1}$$

$$P_0 = \left[\left\{1 \times (\frac{2.4}{9.0})^0 + 1 \times (\frac{2.4}{9.0})^1 + \frac{1}{2!} \times (\frac{2.4}{9.0})^2 + \frac{1}{3!} \times (\frac{2.4}{9.0})^3\right\} + \frac{1}{4!} \times (\frac{2.4}{9.0})^4 \times \frac{4(9.0)}{4(9.0-2.4)}\right]^{-1}$$

$$P_0 = \left[\left\{1 + \frac{2.4}{9.0} + \frac{1}{2} \times \frac{5.76}{81} + \frac{1}{6} \times \frac{13.82}{729}\right\} + \frac{1}{24} \times \frac{33.18}{6561} \times \frac{36}{26.4}\right]^{-1}$$

$$P_0 = [\{1 + 0.27 + 0.036 + 0.0032\} + 0.0069]^{-1}$$

$$P_0 = 0.76$$

Hence, the Probability of zero patients in the system$=$ 0.76

The expected number of patients in the waiting line

$$L_q = \frac{(\frac{\lambda}{\mu})^k}{k!(1-\rho^2)} P_0$$

$$L_q = \frac{(\frac{2.4}{9.0})^4}{4!(1-0.07^2)} \times 0.76$$

$$L_q = \frac{0.0051}{23.88} \times 0.76$$

$$L_q = 0.00016$$

The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$
$$L_s = 0.00016 + \frac{2.4}{9.0} = 0.27$$

The expected waiting time in the queue

$W_q = \frac{L_s}{\lambda} = \frac{0.27}{2.4} = 0.11$ minutes

$= 0.11$ minutes average time a patient spends in the queue waiting

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.11 + \frac{1}{2.4} = 0.53$ minutes average time a patient spends in the system

4.3.2 Analysis of Data Collected on Friday at the Consulting Rooms

Inter arrival rate $= (7 - 3) + (11 - 7) + \cdots + (57 - 54) + (59 - 54)$

$= 4 + 4 + \cdots + 3 + 2$

$= 56$ Minutes

Mean arrival rate $\lambda = \frac{56}{20} = 2.8$ minutes per Patients

Table 7: Number and Time Patients were served at the Consulting Rooms on Friday

Number of Patients	Service Time in Minutes		
	Consulting Room 1	Consulting Room 2	Consulting Room 3
1	7	6	7
2	14	11	26
3	20	15	31
4	31	20	35
5	35	27	44
6	39	31	55
7	45	38	65
8	51	46	78
9	59	51	87
10	61	68	93
11	72	73	99
12	81	87	110
13	89	93	119
14	92	100	
15	99	107	
16	104		
17	116		
18	121		
19	130		
20	132		

Source: Field Data, 2021

Inter service rate (Consulting room 1) $= (14 - 7) + (20 - 14) + \cdots + (130 - 121) + (132 - 130)$

$= 7 + 6 + \cdots + 9 + 2$

$= 125$ Minutes

Mean service rate $\lambda = \frac{125}{20} = 6.3$ minutes per Patients

Inter service rate (Consulting room 2) $= (11 - 6) + (15 - 11) + \cdots + (100 - 93) + (107 - 100)$

$= 5 + 4 + \cdots + 7 + 7$

$= 101$ Minutes

Mean service rate $\lambda = \frac{101}{15} = 6.7$ minutes per Patients

Inter service rate (Consulting room 3) $= (26 - 7) + (31 - 26) + \cdots + (110 - 99) + (119 - 110)$

$= 19 + 5 + \cdots + 11 + 19$

$= 113$Minutes

Mean service rate $\lambda = \frac{112}{13} = 8.6$ minutes per Patients

Mean service rate of the 3 consulting rooms$= \frac{6.3+6.7+8.6}{3} = 7.2$

Number of servers k = 3

Mean combined rate of all servers$= K\mu = 3(7.2) = 21.6$

Utilization factor of the entire system$= \rho = \frac{\lambda}{k\mu} = \frac{2.8}{3\times7.2} = 0.13\ or\ 13\%$

The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left[\left\{\sum_{n=0}^{k-1} \frac{1}{n!}(\frac{\lambda}{\mu})^n\right\} + \frac{1}{k!}(\frac{\lambda}{\mu})^k\left(\frac{k\mu}{k(\mu-\lambda)}\right)\right]^{-1}$$

$$P_0 = \left[\left\{1\times(\frac{2.8}{7.2})^0 + 1\times(\frac{2.8}{7.2})^1 + \frac{1}{2!}\times(\frac{2.8}{7.2})^2\right\} + \frac{1}{3!}\times(\frac{2.8}{7.2})^3\times\frac{3(2.8)}{3(7.2-2.8)}\right]^{-1}$$

$$P_0 = \left[\left\{1 + \frac{2.8}{7.2} + \frac{1}{2}\times\frac{7.84}{51.84}\right\} + \frac{1}{6}\times\frac{21.952}{373.248}\times\frac{8.4}{13.2}\right]^{-1}$$

$$P_0 = [\{1 + 0.39 + 0.076\} + 0.037]^{-1}$$

$$P_0 = 0.67$$

Hence, the Probability of zero patients in the system$= 0.67$

The expected number of patients in the waiting line

$$L_q = \frac{(\frac{\lambda}{\mu})^k}{k!(1-\rho^2)}P_0$$

$$L_q = \frac{(\frac{2.8}{7.2})^3}{3!(1-0.13^2)}\times 0.67$$

$$L_q = \frac{0.059}{5.8986}\times 0.67$$

$$L_q = 0.0067$$

The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = 0.0067 + \frac{2.8}{7.2} = 0.12$$

The expected waiting time in the queue

$W_q = \frac{L_s}{\lambda} = \frac{0.12}{2.8} = 0.043$ minutes

$= 0.043$ minutes average time a patient spends in the queue waiting

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.043 + \frac{1}{2.8} = 0.4$ minutes average time a patient spends in the system

5. Conclusion

Queue analyses were done with the use of observational data obtained from Ahmadu Bello University, Zaria, Kaduna State, Nigeria. The results of the Multi-server-single channel queuing Models reveal that Ahmadau Bello University, Teaching hospital, Zaria has a low utilization factor of 13%. Similarly, the results obtained from questionnaires revealed that patients were generally dissatisfied with service quality in the hospital. This translates in to fact that there is overcrowding of patients in the hospital, usually number of patients outnumbers doctors, nurses and auxiliary staff .This tends to put hospitals' staffs under pressure and hence force them to dispose patients without thorough investigation or treatment which often lead to patients' dissatisfaction. The results also indicated the higher the service quality, the better satisfaction patients derived from the services.

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