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The Concept of Curvature in Fiber Bundles

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Abstract

Many basic concepts in theoretical physics can be interpreted in term of fiber bundles. A fiber bundle is a manifold that locally looks like a product manifold. The well-known examples are the tangent and the cotangent bundles. The aim of this paper is to discuss and explain the concept of curvature in fiber bundles. We followed the analytical induction mathematical method and we found that the curvature is very important concept in fiber bundles.We use Theory of fiber bundles to illustrate the structure homogenous C^*algebras.

Keywords: Concept, Curvature, Fiber Bundles, Manifold.

1. Introduction

Fiber bundles and more general fiberation are basic objects of study in many areas. A fiber bundle with base space B and fiber f can be viewed as a parameterized family of objects each is omorphis of where the family is parameterized by points in B for example a vector bundles over a space B is parameterized of family of vector spaces. The Concept of fiber bundles is absolutely central in contemporary physics. There are two kind of fiber bundles: principal bundles and associate bundles, fiber bundle took place in 1935 _ 1940. [2]

It may be surprising at first sight that $R(x, y)V$ does not depend on any derivatives of need, it seem to tell us less about V than about the connection it self.

Definition (1.1):

Given a liner connection ∇ on a vector bundle $E \to M$ the curvature tensor is the unique multiliner bundle Map $= TM \oplus TM \oplus E \to F(x, y, v) \to R(x, y)v$. Such that for all $x, y \in$ $vec(M) and veT(E)$.
$R(X, Y)V = (\nabla_X, \nabla_Y - \nabla_Y \nabla_X) - \nabla_{(X,Y)V}$[4].

3- The notion of a principal fiber bundle is the appropriate mathematical concept underlying the formulation of gauge theories that provide the general frame work to describe the dynamics of all non-gravitation, interact or, the concept of connection on such principal bundles codifies the physical notion of the bubonic particles mediating. The interaction namely the agaue bosons like the photen, the gulon on the other hand the notion of associated fiber bundles is the apropitematical frame work describe matter fields that interact through the exchange of gauge bosons provisional definition.

2. A Fiber Bundle Consist at Least of the Following:

i. A topological spaces B called the bundle space.
ii. A topological Space X called the base space.
iii. A continuous Map $P: B \rightarrow X$

of B onto X called the projection and (Lv) a space y called the fibre the set y defined by $y = P^{-1}(X)$

is called the fibre over the point x of x it is required that each y be homomorphic to y .Finally for each of x there is a nighboyhood v of x and ahomemorphism

$$\emptyset: Vxy \rightarrow P^{-1}(V)$$

Such that

$$P\emptyset(x, y) = x\overline{X} \in V$$
$$y \in y$$

A cross of a bundle is continuous Map: $X \rightarrow B$

Such that

$Pf(x) = x \text{ for each } x \in X$

The above definition of bundle is not sufficiently restrictive A bundle will be required to carry additional structure in evolving a group G homeomorphism of y called the group of the bundle [6]

A proper equivalence relation follows quickly. Reflexivity is immediate. Symmetry follows from the continuity of $(g_1, g_2) \rightarrow g_1 g_2$ equivalence class of coordinate bundles.

One may regard afibre bundle as maximal coordinate bundles having all possible coordinate function of an equivalence class. As our indexing sets are unrestricted. This involves the usual logical difficulty connected with the use of the word all.

3. Mappings of Bundles:

Let $\overline{\emptyset}$ be two coordiate bundles having the same filres and the same group map. By a maph: $\theta \rightarrow \overline{\theta}$is meant a continuous map $h: B \rightarrow \overline{B}$ having the following properties.

A carries each fibrey of B homcomorphically onto a fibrey of $\overline{B}$ thusinducing a continuous map $h: x \rightarrow \overline{x}$ such that

$$\overline{p}h = \overline{h}p \qquad (1)$$

if$x \epsilon v_i \cap h^{-1}(\overline{v}_k)$ and$h_x: y_x \rightarrow y_{\overline{x}}$is the map induced by$h(\overline{x}) = \overline{h}(x)$then the map

$$g_k(x) = \emptyset_{kj}^{1-1} h_x^{2\emptyset jk} = P_k h_x \emptyset_{j,x} \qquad (2)$$

In the literature the map h is called Fibre preserve we shall use frequently the expression bundle map to emphasize that his map in the above sense. It is readily proved that the identity map $B \rightarrow B$ in this sense. Likewise the composition of two maps $B \rightarrow B' - B''$ is also map B– B".

A map of frequent occurrence is an inclusion map$\beta C \bar{\beta}$ obtained as follows. Let $\bar{\beta}$ be a coordinate bundle over $\bar{x}$ and letx be subspace of $\bar{x}$ let $\beta = p^{1-1}(x), P \equiv \bar{p} \setminus B$ (1.3.4) and define the coordinate functions of $\beta y \emptyset_i(\bar{v} \cap x)xy$. Then β is a coordinate functions ofβ by$\emptyset_i = \bar{\emptyset}_i(\bar{v} \cap x)xy$. Then β is a coordinate bundle, and the inclusion map $\beta \rightarrow \bar{\beta}$ is map$\beta - \bar{\beta}$. we call β the portion of $\bar{\beta}$ over x or β is $\bar{\beta}$restricted to x and we will use the notations.

$$\bar{g}_{kj} \text{of } \beta = \beta_i = \bar{\beta} \quad (3)$$

The functions and are called the mapping transformation. There are two sets of relation which they satisfy

$$\bar{g}_{kj}(x) g_{ji} = \bar{g}_{kj}(x), x \in v_i \cap v_j \cap h^{-1}(\bar{v}_k) \; (4)$$

$$\bar{g}_{ik}\left(\bar{h}(x)\bar{g}_{ik}(x)\right) = \bar{g}_{ij}(x)c \in v_j \cap h^{-1}(\bar{v}_k \cap \bar{v}_i) \; (5)$$

Lemma (3.1):

Let $\beta, \bar{\beta}$ be Coordinate bundies having the same fiber y and group G and let $\bar{h}: x \rightarrow \bar{x}$ be a map of one base space into the other finally let$\bar{g}_{kj}: v_j \cap \bar{h}^{-1}(\bar{v}_k) \rightarrow G$ be a set of continuous maps satisfying the conditions (1.3). Then there exists one only one map $h: \beta \rightarrow \bar{\beta}$ inducing h and having$\{\bar{g}_{jk}\}$ as its mapping transformation.

$$\text{if } p(b) = x \text{ lies in } v_j \cap h^{-1}(\bar{v}_k) \quad (6)$$

$$h_{ki}(b) = \bar{\emptyset}_k\left(\bar{h}(x)\bar{g}(k), p(b)\right) \quad (7)$$

Thenh_{ki} is continuous in b and $\bar{p}h_k(b) = \bar{h}(p(b))$ suppose$x \in v_i \cap v_i \cap \bar{h}^{-1}(\bar{v}_k \cap \bar{v}_i)$, we have$(with \bar{x} = \bar{h}(x))$

$$h_{kj}(b) = \bar{\emptyset}_k(\bar{x}), \bar{g}_{ki}(x) g_{ji}(x) p_i(b) \quad (8)$$

$$= \bar{\emptyset}_k(\bar{x}, \bar{g}_{ki}(x)\, p(b) - h_{ki}(b) \quad (9)$$

$$= \bar{\emptyset}_i(\bar{x}, \bar{g}_{ik}(\bar{x})\, \bar{g}_{ki}(b) - p_i(b)(10)$$

$$= \bar{\emptyset}_k(\bar{x}, \bar{g}_{ki}(x) p_i(b) = h_{ji}(b) \; (11)$$

It follows that any two functions of the collection $\{h_{kj}\}$ agree their common domain. Since their domain are open and coverβ , they define a single valued continuous function$h$. Then $\bar{p}h = \bar{h}p$ follows from the relation for h_{ik}. If we replace b by $\emptyset_i(y)$ apply $\bar{P}_k$ to both sides, and use the relations we obtain $\bar{P}_k h\emptyset_i(y) = \bar{P}_k\bar{\emptyset}_k\left(\bar{x}, \bar{P}_{kj}(x)P_i\emptyset_i, \pi(y)\right) = \bar{g}_k(x)y$ (12)

Which shows that the described mapping transformations

Lemma (3.2):

Let $\beta, \bar{\beta}$ be Coordinate bundles having the same fiber and group and let $\bar{h}: \beta \to \bar{\beta}$ be a map such that the induced map $\bar{h}: x - \bar{x}$ is 1-1 and has a continuous inverse $\bar{h}^{-1}: x \to \bar{x}$. Then h has h has a continuous inverse $\bar{h}^{-1}: \bar{\beta} \to \beta$ and $\bar{h}$ is map $\beta \to \bar{\beta}$. The fact that h is 1-1 in the large is evident for $\bar{x}$ in $\bar{v}_k \cap \bar{h}\,(v_i), Let x\ = \bar{h}^{-1}(\bar{x})$ and following defines.

$$\bar{g}(\bar{x})\emptyset_{jix}^{-1}h_k^{-1}\emptyset_{k\bar{x}} \quad (13)$$

It follows that $\bar{g}(\bar{x}) = \bar{g}(x)^{-1}$ since $g \to g^{-1}$ is continuous in G, x is continuous in $\bar{x}$, and $\bar{g}_{kj}(x)$ is continuous x. it follows in that $\bar{g}_{kj}(\bar{x})$ is continuous in $\bar{x}$, if $\bar{p}(\bar{b}) = x$ is in $\bar{v}_k \cap h(v_j)$ then h^{-1} is given by $h^{-1}(\bar{b}) = \emptyset_i\,(\bar{h}^{-1}(\bar{x}),\ \bar{g}_{jk}\,(\bar{x}), \bar{P}_k\,(\bar{b}))$ which shows that $h^{-1}$is continuous on $\bar{p}(\bar{v}_k \cap h(v_j))$ since these sets are open and cover it follows that $\bar{h}$is continuous and the Lemma is proved.

Two coordinate bundle β and $\bar{\beta}$having the same base space, fiber and group are said to be equivalent if there exists

a map $\beta \to \bar{\beta}$ which induces the identity map of the common bases space. The symmetry of this relation is provided by the above lemma. The reflexivity and transitivity are immediate It is to be noted that strict equivalence defined in implies equivalence Two fiber bundles having the same base space-fiber and group are said to be equivalent if They representative coordinate bundles which are equivalent.

It is possible to define b roade notions of equivalences of fiber bundles. by allowing x or (y, G) to vary by topological equivalence.

The effect of this is to reduce the number of equivalence classes. The definition chosen is the one most suitable for the classification theorems proved later.

Let $\beta, \bar{\beta}$ be coordinate bundles having the same base space, fiber, and group, then they are equivalent if and only if there exist continuous maps.

$$\bar{g}_{kj}: v_i \cap \bar{v}_k \to G i, j, k \in J$$

$$\bar{g}_{ki}(x) = \bar{g}_{ki}(x)g_{ji}(x) \quad (1.3.16) \qquad x \in v_i \cap v_j \cap \bar{v}_k$$

$$\bar{g}_{ij}(x) = \bar{g}_{ik}(x)g_{ki}(x) \quad (1.3.17) \qquad x \in v_i \cap \bar{v}_k \cap \bar{v}_i$$

Suppose first that $\beta, \bar{\beta}$ are equivalent and $h: \beta \to \bar{\beta}$ define $\bar{g}_{kj}$by (1.2) (note that $\bar{x} = x$ since h is the identity). Therelation (1.5) se reduce to (1.5)

Conversely suppose, the$\bar{g}_{kj}$are given. The relations (1.5) imply (l.5) in the case $\bar{h}\ =$ identity. The existence of h is provided Let β be a coordinate bundle with neighborhood $\{v\}$ and let $\{\bar{v}_k\}$ be a covering of x by an indexed family of open sets such that each$\bar{v}_k$is contained in some v_i(i.e. the second covering is a refinement of the first) then one constructs a strictly equivalent coordinate bundle $\bar{\beta}$with neighborhoodsv_k by simply restricting $g\emptyset_j$ to $\bar{v}_k xy$ where$j$ is selected so that $\bar{v}_k \subset v_i$ so when j, k are so related the function $\bar{g}_{kj}$of (15) are constant and equal the identity of G.

Suppose now $\beta, \bar{\beta}$. are two Coordinate bundles with the same base space fiber and group. The open sets$v_j\ \cap \bar{v}_k, j\ \in J, k\ \in \bar{J}$ cover x and form a refinement of $\{v_j\}$ and $\{\bar{y}\}$It follows that, $\bar{\beta}$, are strictly equivalent to coordinate bundles $\beta, \bar{\beta}$of respectively having the same set of coordinate neighborhoods. This observation lends weight to the following lemma.

Lemma (3.3):

Let $\beta, \bar{\beta}$two coordinate bundies with the same base space fiber, group and coordinate neighborhood let $g_{ij}, \bar{g}_{ji}$ denote their coordinate transformations. Then $\beta, \bar{\beta}$, are equivalent if and only if there exist continuous function $\lambda\Gamma_i: v_i - G$ defined for each j and J and such that

$$\bar{g}_{ji}\ (x)\ = \lambda_i(x)^{-1} g_{ij}(x) \lambda_i\ (x) \quad (1.3.8) \quad x \in v_i\ \cap v_j$$

if$\beta, \bar{\beta}$ are equivalent the function g_{kj} provided enable us to define$\lambda_i\ (\ \bar{g}_{ji}\ )$ Then relations (1.5) yield (1.5).

Conversely, suppose theλ_i satisfying (1.5) are given define

$$\bar{g}_{ki}\ (x)\ = \lambda_{\mathrm{k}}(x)\ ^{-1} g_{kj}(x) \qquad x \in v_j\ \cap v_k$$

Then the relation (1.5) follow from (1.5) and lemma is proved.

Definition (3.4):

A Generalc^r bundle is a triple $\varepsilon =\ (E, \pi, x)$ where $\pi: E\ \rightarrow\ M$ is a surjective c^rmap of c^rspaces (called the bundle projection) for each $p\ \in\ x$the subspace $E_p: \pi^{-1}(p)$ is called the total space and x is the base space if $s \subset x$ is a subspaces we can always form the restricted bundle (E_s, π_s, s) where $E_s = \pi^{-\ 1}(s)$ and $\pi_s = \pi\ / s$ is the restriction.

4. cA (c^r −) Section of a General Bundle:

$\pi_E: E \rightarrow M$is a (c^r) maps: $M \rightarrow\ E$ such that $t\pi Eos\ =\ id$M.

In other words, the following diagram must commute.

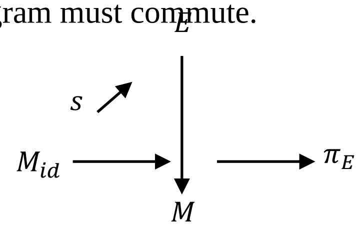

Fig (1)

The set of all c^rsections of general bundle $\pi_E: E \rightarrow M$is denoted by $T^k\ (M, E)$. We also define the notion of a sectionover an open set U in M is the obvious way and these are de noted by $T^k\ (U, E)$.

Notation (4.1):

We shall often abbreviate to just T(U. E) or even T(E) whenever confusion is unlikely. This is especially true in case $k\ = \infty$(smooth case) or k = 0 (continuous case).

Now there are two different ways to treat bundles as a category.

The category Buna, Actually, we should define the Categories Burk: $k\ =\ 0{,}1\ldots\ldots\infty$ and Then abbreviate to gust "Bun" in cases where a context has been establish and confusion is unlikely. The objects! of bunk are c^rfiber bundle.

Definition (4.2):

Amorphism from Hom$\beta(\varepsilon_1, \varepsilon_2)$ also called a bundle map from a c^r fiber bundle $\varepsilon_1(E_1, \pi_1, x_1)$ to another Fiber bundle $\varepsilon_2(E_2, \pi_2, x_2)$ is a pair of c^rmaps $(\bar{f}, f)$ such that the following diagram commutes

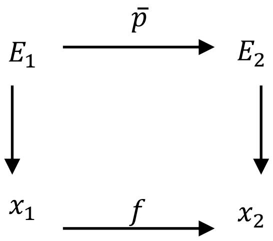

Fig (2)

If both maps are c^r isomorphism's we call the map a(c^r) bundle isomorphism over x (called bundle equivalence).

5. Two Fiber Bundles:

$\varepsilon_1\ (E^1, \pi^1, x)$ and $\varepsilon_2\ (E_2, \pi_2, x_2)$ are equivalent in Bunk (x) or isomorphic if there exists a (c^r) bundle isomorphism over x from x from $\varepsilon_1 to \varepsilon_2$.

By now the reader is no doubt tired of the repetitive use of the index c^rso from now on we will simplifier to space or (manifolds) and maps where the appropriate smoothness c^r will not explicitly stated unless something only works for a specific value of$r$.

Definition (5.1):

Two fiber bundles $\varepsilon_1\ (E_1, \pi_1, x)$ and $\varepsilon_2(E_2, \pi_2, x)$ are said to be locally equivalent (over x) if for any $y\ \in E$ there is an open set u containing p and a bundle equivalence $(\bar{f}.f)$ of the restricted bundles:

$$E_1/u \xrightarrow{\bar{f}} E_2/u$$

$$u \xrightarrow{id} u$$

Now for any space x the trivial bundle with fiber of is the! Triple$(Xxf.pr,x)$ where pr_1, always denoted the projection onto the first factor any bundle over$x$ that is bundle equivalent. To $xxf$is referred to as trivial bundle, we will now add in an extra condition that will usual need.

A (-) Fiber Bundle (5.2):

ε: (E,π ,x) is said to be locally trivial (with fiber f) if every for every $x \in x$ has on open neighborhood u such thatεu: $(Ev, \pi u, v)$ is isomorphism to the trivial bundle $(u \times f.pr_1, u)$ a such a Fibre bundle is called alocally trivial Fiber bundle we immediately make the following convention All fiber bundles in the book will be assumed to be locally trivial unless otherwise stated once we have the local triviality it follow that each fiber $Ep = \pi^{-1}\,(p)$ is homomorphic (in factdiffeomorphic) to f.

Notation (5.3):

We shall take the liberty of using a variety of notions when talking about bundles most of which are quite common and so the reader may as well get used to them one perhaps less common, notation which is very suggestive is writing $\varepsilon(f \to E - x)$ to refer to fiber bundal with typical fiber$f$.

The natation suggests that f may be embedded into E as one of the fibers. This embedding is not canonical in general.

It follows from the definition that there is a cover of E by bundle charts or (local) trivializing maps $f_\propto$: $E_{j\propto} \to u_\propto x f$such that

$$Eu_\propto \xrightarrow{\bar{f}} u_\propto f$$

$$u_\propto$$

Fig (3)

Which in turn means that is called overlap maps$f_o f_\beta^{-1}$: $U_\propto \cap\cup xf \to$ must have the form $f_\propto^{-1} 0 f_\beta^{-1}(x,u) = (x, f_{\beta\propto}, x(u))$ for maps x. $f_{\beta\propto,x} \in D_i f f^r$(ƒ)defined on each non-empty in trisection $U_\propto \cap U$. There are called transition functions and will assume them to be maps whenever there is enough structure around to make sense of the notion such a Cover by bundle Charts is called a bundles at Las a c^r.

Definition (5.4):

It may be that there exists a group G (group in the smooth ease) and are presentation p of G in $Diff^r\,(f)$ such that for each non-empty $U_\propto \cap U_\beta$, we have $f_{\beta\propto,x} = p(g_{\beta\propto}(x))$ for

Some c^r map $g_{\beta\alpha}: U_\beta \to G$, in this case we say that G serves as a structure group for the bundle via the representation p In case the representation is a faith one then we may take G be sub group of $D_iff^r\ (F)$ and then we simply have$f_{\beta\propto,x} = g_{\propto\beta}(x)$. Alternatively we may speak in term or group action so that G acts on f by diffeomorphisms.

The maps $g_{\propto\beta}: U_\propto \cap U_\beta \to G$ must satisfy certain consistency relations.

$$g_{\propto\propto}(x) = idforx \in U_\propto$$

$$g_{\propto\beta}(x) g_{\beta\propto}(x) = idforx \in U_\propto \cap U_{\beta\propto}$$

$g_{\propto\beta}(x) g_{\beta y}(x) g_{y\propto}(x) = idforx \in U_\propto \cap U_\beta \cap U_y$ [38]

Lemma (5.5):

Let $B, \bar{B}$ be two coordinate bundles the same fibre and group and let h be a map $B - \bar{B}$ coresponding to each cross section $\bar{f}: \bar{X} \to \bar{B}$there exists one and only one cross section $f: X \to B$ such that $hf(x) = \bar{f}h(x) \quad x \in x$.

The cross-section f is said to be induced by $h$and $\bar{f}$, and will be denoted by $\bar{h}\bar{f}$.

Let $\bar{x} = \bar{h}(x)$ Since $f(x)$ must lie in y and $h_x: y_x \to y_x$ is l-1 map, it follows that$f(x) = h_x^{-1}(\bar{x})$.

This defines f and proves its uniqueness. It remainsprove continuity It suffices to show that Fiscontinuous over any set of the form $N \cap h^{-1}(\bar{v}_k)$ for these sets are open and cover x. Since$pf(x) = x$ is continuous. it remains to show that $pf\ (x)$ is continuous. By (18)$\bar{g}_{ki}\ (x)$ is continuous furthermore

$$\emptyset_{kj}, \frac{x}{P_{xj}}, f(x) = \emptyset^{-1} h_x \emptyset_j f(x) = \emptyset^{1-1}_{h(x)} hxf(x)$$

Therefore

$$\emptyset^{1-1}_{k(x)} f\left(\bar{h}(x)\right) = f(\bar{h}(x))$$

$$= p_i f(x) = [g_{ki}(x)]^{-1}$$

Is also continuous.

The Lemma shows that cross sections behave contra valiantly under mapping of bundles in this respect they resemble covariant tensors.

Point Set Properties of B (5.6):

It is well known that numerous topological properties of$x$ and$y$ carry over to their product xxy. They also carry over to the bundle space B of any bundle with base space x and fibre y. The argument given for the product space carries over to the bundle using the local product representations given by coordinate function.

As an examples suppose x and$y$ are Hausdorff spaces. Let $b, \bar{b}$ be distinct points of B. If $p(\sigma) \neq p(\sigma)$ Let u, y be neighborhoods of $p(\sigma), p(\sigma)$ such that $u \cap v = 0$. Then $p^{-1}(u)$ and $p^{-1}(v)$ are non-overlapping open sets containing$\sigma$and $\bar{\sigma}$. if $p(\sigma) = p(\sigma) = x$. Then choose aj such that x is in v. Now $p_j(\sigma) \neq p(\sigma)$ since therefore there exist neigh book hoods $u, \bar{u}$ of $p_j(b)p_j\ (\bar{b})$ such that it $u \cap \bar{v} = 0$. Then $\emptyset_j(v_j \times u)$ and $\emptyset_j(v_j \times \bar{u})$ are non-overlapping open sets and containing band $\bar{b}$. Thus B is a Hausdorff space.

As a second examples suppose x and y are compact Hausdorff spaces. For each point x, choose $a_j$such that x is in v_j and choose an open set u_x, such that x is in u_x, and$u_x \subset v_x$. (this can be done since any compact Hausdorff space is regular). The sets $[\mathrm{U_x}]$ cover x; select a finite covering $\mathrm{u_i}, \ldots\ldots, \mathrm{u_m}$SinceU$_{\mathrm{x}}$ is compact so is $\overline{\mathrm{U}}_{\mathrm{x}} \times \mathrm{y}$ select; so that $\mathrm{U_x} \subset \mathrm{v_j}$ since $\emptyset_{\mathrm{j}}$, is a topological map, it follow that $\mathrm{p}^{-1}(\mathrm{u_r})$ is compact. But these sets for $\mathrm{r} = 1, \ldots, \mathrm{m}$ cover the space B there fore B is compact.

Among other common properties of x and y which are also properties of B we mention:

(i) Connectedness. (ii) The first axiom of accountability. (iii) existence of a countable base for open sets.(iv) Local compactness. (v) local connected ness and (vi) are wise connected ness.

6. Construtton of A Bundle From Coordinate – Transformations:

Let G be a topological group, and x a space. By a system of coordinate transformations in x with values in G is meant an indexed covering $\{v_i\}$of $x$by open sets and a collection of continuous maps

(A) $g_{ji} v_i \cap v_j \to G$

such that

$$g_{ki}(x) g_j(x) = g_{ki}(x) \quad x \in v_i \cap v_k \ (1.3.20)$$

The relations $g_{kii}\ (x) = e\ and\ g_{ij}(x) = (g_{ji}(x))^{-1}$

We have seen in 3,3 that any bundle over x with group G determines such a set of coordinate transformation, we shall prove a converse.

7. Curvature

If E → M is a vector bundle and ∇ is a liner connection, it is a natural to ask whether covariant derivative operator ∇_{x} and ∇_{y} is different direction commute. Of course this is not even true in general for the lie derivatives $\mathrm{L_x}$ and $\mathrm{L_y}$on $\mathrm{C}^{\infty}(\mathrm{M})$.

Which can view as the trivial connection on a trivial line bundle. Their Lake of commutability can however be measured via the identity $L_x L_y - L_y L_x = L_{(x,y)}$ [1]

8. Results

We found the following. Results: The theory of fiber bundles illustrate the structure of homogenous C^*algebras, the sets of C^*algebras are compact and many important application of manifold theory, volumes are computed by integration and curvature typically. The most important fiber bundles are vector bundles and principal bundles.

9. Conclusion

The expression bundle mean coordinate bundle, Fiber bundles not be the primary, but rather the ultimate objects of study will be studied through their representative. The various concepts introduced for coordinate bundles must be have property equivalence. The situation is similar to that group the only when one studies group G given by generators and relation. Results which are not invariant under a change of base are of little interest.

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