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Multiobjective Optimization of Investments of two Businesses in Ghana

Jones Akanyare[1] and Stephen B. Twum[2]
[1]Department of Mathematics, University for Development Studies, Tamale, Ghana
[2]Department of Mathematics, C. K. Tedam University of Technology & Applied Sciences, **Ghana**
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Abstract

Portfolio optimization is a major concern of individuals and businesses across the world for sustainable financial and economic management of their funds. This work, as a first part of a portfolio optimization study of two selected businesses in Ghana, is aimed at optimizing separately the investments of the two businesses, to maximize their returns and minimize their risks under their operational constraints. The problems, being multi-objective in character, were modeled with the expected returns on their investments and the variances/standard deviations of the returns over fixed periods of time as the objective functions. To ensure dimensional uniformity, the objective functions were normalized and the Weighted Sum scalarization method employed in MATLAB to find Pareto optimal solutions of the models, using data from the two businesses. The results reveal that weight variations do not necessarily lead to many varied or diversified Pareto optimal solutions. The three distinct Pareto optimal solutions obtained for one of the businesses, however, suggest that the business could make respectively about 104.45M, 15.70M, and 15.72M Ghana Cedis returns on its investments with risk to return margins of 0.225%, 1.77%, and 1.76% respectively. The other business with only two distinct Pareto optimal solutions could make 14.67M or 30.36M Ghana Cedis returns with corresponding risk to return margins of 0.016% and 0.017% respectively. It is recommended therefore that the businesses could select the solution with the least risk to return margins for implementation. A second part of the work, which will be reported in another paper, would investigate the models under post-optimality analysis and the prospects for a joint investment by the two businesses.

Keywords: Bi-criteria Optimization, Investment, Risk, Return, Standard Deviation, Pareto Optimal Solution.

1. Introduction

One of the problems faced by investors is deciding on the amounts to allocate to available investment portfolios in order to maximize the return while simultaneously minimizing the risk of the investments over a given period (Qu et al, 2017). This is a very crucial decision-making task that investors and portfolio managers alike have to address appropriately in order to ensure the growth and sustainability of their businesses.

Portfolio investment problems, since the ground-breaking work of Harry Markowitz (Markowitz, 1952), have traditionally been treated as single objective optimization problems where either risk is minimized or expected return is maximized. Makowitz's work forms the foundation of modern Portfolio theory. He used the model to describe the impact of Portfolio diversification by the number of securities within the Portfolio and their covariance relationship (Souza & Megginson, 1999). Markowitz states that, the expected return (Mean) and the risk (Variance or standard deviation of the expected return) of investments are the main criteria for portfolio selection and construction (Markowitz, 1959). Despite the fact that the Markowitz Model takes a narrow view, which is that it is premised on optimizing a single objective, it is undisputed that it is the most widely used model by researchers and practitioners in real world applications (Fama & French, 2004).

Typically, an investor may seek to select from available investment portfolios which yield maximum return or profit, or may want to decide on the amounts to invest in known (or existing) investment portfolios in order to maximize the total return, or may want to decide on a combination of both types. These three types of portfolio investments decision making are optimization problems (Kolm et al, 2014), and have been approached in diverse ways as such, using mathematical programming techniques (Bagchi, 2014; Keshavarz & Toloo, 2015). Depending on the nature of the problem and the interests or focus of the decision maker, the formulated mathematical programming models may be linear, nonlinear, integer programming, mixed integer programming, or multi-criteria programming. Even so, a large part of the literature in the subject area appears to indicate that a single objective formulation in which return alone or risk alone are optimized, subject to identified constraints of the problem, dominate. Comparatively fewer works have approached the problem as multi-criteria programming. Nevertheless, the multi-criteria approach is a more realistic one since it seeks to account for all the relevant criteria or goals (not just one), and therefore, provides greater utility to the investor or would-be investor (Ponsich et al, 2013; Skolpadungket et al, 2007; Vedarajan,1997).

Some notable research works in this study area include Kamil & Kwan (2004) who applied the Markowitz model for portfolio analysis on assessing the performance of selected stocks from the stock market. The work suggests that long term (weekly) investments are more likely to perform better than short term (daily) investment, with the same Portfolio. In the daily analysis of data for the Portfolio, a yield of a small positive covariance was made but, a weekly analysis of the same data yielded a negative covariance, which is good for businesses. The work only considered analyzing the performance of selected stocks. It did not take interest in maximizing returns or minimizing risks for the investor. In a model proposed by Wagner (2002), the decision maker is provided with the opportunity to access an ex-ante idea about a Portfolio selection. This efficiently helps reduce regrets the decision maker could have faced. The paper incorporated a rational portfolio selection criteria as well as benchmarking. Pandey (2012) worked on optimal Portfolio formation, using real data, subject to different constraint sets. The concept of efficient Portfolio formed the basis for decision making. Given a level of risk, the highest Portfolio return was obtained from the efficient frontier. On the risk-return curve, the investor is able to have a reliable insight into how to efficiently manage Portfolio selection in order to reduce risk or maximize returns. Miettinen & Mäkelä (2002) conducted an extensive work on multi-objective optimization of portfolio. In their work, they were only concerned with solution methods to multi-objective portfolio optimization problems. To reduce the complexity of large-scale portfolio optimization, Qu et al (2017) proposed two asset preselection procedures that consider return and risk of individual asset and pairwise correlation to remove assets that may not potentially be selected into any portfolio. To test the effectiveness of the proposed methods, a Normalized Multiobjective Evolutionary Algorithm based on a decomposition algorithm and several other commonly used multiobjective evolutionary algorithms were applied and compared.

This paper contributes to the literature in the subject area using the multi-criteria approach in the context of real investment problems involving two businesses in Ghana and based on real data from them. Specifically a bi-objective optimization model is formulated to simultaneously optimize risk and return on their investments in terms of the amounts of funds to invest in existing investment sreas. .The next section presents the methodology adopted in this work. The next section after that presents the results and discussions of the research. The last section concludes the paper and presents some recommendations.

2. Methodology

2.1 Multicriteria Optimization

In the context of constrained optimization, Multi-criteria optimization is concerned with optimizing simultaneously two or more objective functions, subject to a number of constraints (Marler & Arora, 2004). Without loss of generality, the problem is denoted by:

$$Minimise\ f(x) = [f_1(x), f_2(x), \dots, f_k(x)]$$

$$\text{Subject to} \quad x \in X \subset R^n$$

(2.1)

where f_i is the i^{th} objective function ($i = 1,2, \dots, k$), x is an n vector of decision alternatives and X is the set of feasible decision alternatives, also called feasible decision set, in which all the constraints are satisfied. The vector function $f(x)$ defines a criterion set in the space R^k from which points in the feasible decision set are mapped (Deb 2001; Miettinen & Makela, 2002). Unlike single objective problems, (2.1) is generally characterized by conflicting or incommensurable criteria and therefore by many solutions (Miettinen, 2000). As a result, the notion of optimality as it is known in single objective optimization becomes untenable and requires re-definition (Miettinen, 2000).

There are several notions of optimality, such as, Pareto optimality, Lexicographic optimality, Min-max optimality and Lexicographic Min-max optimality (Ehrgott, 2008; Marler & Arora, 2004), the most popular, however, is Pareto optimality which is very much related to the concept of dominance and defined as:

A solution $x^* \in X$ *is said to be Pareto optimal (or non-dominated)*
if $f(x^*) \leq f(x)$ *for all* $x \in X$ *and there exists* $x \in X$ *such that* $f(x^*) < f(x)$.
(2.2)

Other notions of Pareto optimality are *Strong*, *Weak* and *Proper* Pareto optimality. They are defined respectively as:

A solution $x^* \in X$ *is said to be strongly Pareto optimal if* $f(x^*) < f(x)$ *for all* $x \in X$
(2.3)
A solution $x^* \in X$ *is said to be weakly Pareto optimal if* $f(x^*) \leq f(x)$ *for all* $x \in X$
(2.4)
A solution $x^* \in X$ *is said to be Properly Pareto optimal if it is Pareto optimal and there exists a positive number* δ *such that for all* $f_i(x)$ *and* $x \in X$ *satisfying* $f_i(x) < f_i(x^*)$, *there exists* $f_j(x) > f_j(x^*)$
such that:

$$\frac{f_i(x^*) - f_i(x)}{f_j(x) - f_j(x^*)} \leq \delta$$

(2.5)

It is clear from (2.3) and (2.4) that strong and weak Pareto optimality are special cases of Pareto optimality as defined in (2.2) with the two notions being particular instances of it. Proper Pareto optimality as given in (2.5) on the other hand defines a limit or bound on the Pareto optimal solutions (Ehrgott, 2008). The set of all Pareto optimal solutions in the feasible decision set and its corresponding image in the criterion set is sometimes called efficient set, non-inferior set, or non-dominated set. The Pareto optimal solutions in the criterion set constitute the Pareto front (Ehrgott, 2008).

Since there is no unique solution to the problem (2.1) but rather many equally good solutions (i.e. Non-dominated solutions), the decision about which one to select for implementation becomes subjective and depends very much on the preferences of the decision maker as it does the analyst (Miettinen, 2000). There is therefore always some trade-offs in the values of the objective functions that have to be incurred by a decision maker in selecting any of the Pareto optimal solutions. The chosen solution is therefore referred to as Compromise Solution (Li & Zhang, 2009).

2.2 Solution Method

Many methods of solution have been developed for solving (2.1) and the choice of a particular method depends on the nature of the problem as well as the expectations of the analyst and decision maker. In general the methods can be classified as Scalar or Pareto (Deb, 2001). Whereas the scalar methods are suitable for continuous, differentiable and deterministic problems the Pareto methods involve use of heuristic or meta-heuristic algorithms and are suitable for cases that depart from the described (Deb, 2001). Classically, the weighted Sum method is the most popular scalar method. Other scalar methods are the constraint method, the global criterion method, and goal programming among many others (see Marler and Arora, 2010).

The weighted sum method, as the name suggests, transforms the vector objective function in a given problem into a scalar one in a convex combination of the objective functions using weights (Marler & Arora, 2010). It is defined by:

$$minimize\ f(x) = w_1 f_1(x) + w_2 f_2(x) + \cdots + w_k f_k(x)$$

(2.6)

Subject to $x \in X$ and $w_1 + w_2 + \cdots + w_k = 1,\quad w_i > 0\quad \forall\, i$

where w_i is the weight of the i^{th} objective function $i = 1,2,\ldots,k$.When w_i is varied, it produces a different scalar problem as far as the weights are concerned, and yields corresponding Pareto optimal solutions (Marler and Arora, 2010). Consequently, the entire set of Pareto optimal solution may be generated .Though this method works so well with convex problems, it is inefficient with non-convex ones as well as problems with many objective functions (Marler and Arora, 2010). It requires little or no input from the user.

2.2 Model Formulation

Consider that a fixed amount of money, M, is to be invested in n known investment portfolios, each of which has a known history of return. Let the problem be to decide on the amounts to put in each investment area so that the total return on the investments is maximized while the total variability in future payments is minimized. Let x_i $(i = 1,2,\ldots,n)$ be the amount of money to put in the i^{th} investment and let r_{ik} be the rate of return on investment i in the time period k in the past $(k = 1,2,\ldots,p)$. If the past history of payments is indicative of future performance, then the expected future return per unit currency from investment i is given by

$$E_i = \frac{1}{p}\sum_{k=1}^{p} r_{ik}$$

(2.7)

The expected return from all the investments therefore is:

$$E = \sum_{i=1}^{n} E_i x_i$$

(2.8)

A measure of total variability in future payments based on past returns is given by

$$Z = \frac{1}{p}\sum_{k=1}^{p}(r_{1k}x_1 + r_{2k}x_2 + \cdots + r_{nk}x_n - E)^2$$

(2.9)

Which is the average over the past p time periods over the squared deviation between the total return from an allocation $(x_1, x_2, \ldots, x_n)$ and the expected value of the total return.

The result (2.9) in statistical terms is the variance of the total returns. Substituting (2.8) in (2.9) and rearranging yields:

$$Z = \frac{1}{p}[(r_{ik} - E_1)x_1 + (r_{2k} - E_2)x_2 + \cdots + (r_{nk} - E_n)x_n]^2$$
$$= \sum_{k=1}^{p}\sum_{i=1}^{n}\sum_{j=1}^{n}(r_{ik} - E_i)(r_{jk} - E_j)x_i x_j = \sum_{i=1}^{n}\sum_{j=1}^{n}\delta_{ij}^2 x_i x_j$$

Where:

$$\delta_{ij}^2 = \sum_{k=1}^{p}(r_{ik} - E_i)(r_{jk} - E_j) = \frac{1}{p}\sum_{k=1}^{p} r_{ik}r_{jk} - \frac{1}{p^2}(\sum_{k=1}^{p} r_{ik})(\sum_{k=1}^{p} r_{jk})$$

is a covariance matrix (i.e. $C \equiv [\delta_{ij}^2]$). We may denote therefore:

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\delta_{ij}^2 x_i x_j = X^T C X$$

(2.10)

Where C is a covariance matrix and $X = (x_1, x_2, \ldots, x_n)^T$. The Portfolio optimization problem is therefore modeled as:

$$maximize\ E(X) = \sum_{i=1}^{n} E_i x_i$$
$$minimize\ Z(X) = \sum_{i=1}^{n}\sum_{j=1}^{n}\delta_{ij}^2 x_i x_j$$
$$Subject\ to\ \sum_{i=1}^{n} x_i \le M$$
$$x_i \ge 0\ i = 1,2, \ldots, n$$

(2.11)

It is observed that $E(X)$ is weakly convex, since it is linear. Also $Z(X)$ is in quadratic form, and so it is convex. The constraint $\sum_{i=1}^{n} x_i$ is linear for all i and hence weakly convex. Therefore, (2.11) is a bi-criteria convex optimization problem in the class of continuous differentiable constrained convex multi-criteria optimization problem. Consequently, it can be solved efficiently by any scalar method, such as the Weighted Sum Scalarization. Therefore, the weighted sum method corresponding to the model (2.11) is given by:

$$Max\ [w_i E(X) - (1 - w_i)(Z(X))]$$

Subject to $\sum_{i=1}^{n} x_i = M$ $i = 1,2, \ldots, n$

(2.12)

$$x_i \ge 0\ \forall\ i,\ w_j > 0, \sum w_j = 1,\ j = 1,2, \ldots . n$$

As a generating or posteriori method (Mosavi, 2010), (2.12) may be used to generate a portion of the Pareto front through repeated sets of weights variations without the involvement of the decision maker; the solutions can then be presented to the decision maker to select the best compromise solution.

Transformation techniques are sometimes used to normalize the objective functions to make them dimensionless so as to enable appropriate comparison of their values in the solution search process. A notable transformation approach proposed by Proos et al (2001) to be used in this work is:

$$f_i^{norm} = \frac{f_i(x)}{|f_i^{opt}|} \quad , \qquad 0 \le f_i^{norm} \le 1, \qquad i = 1,2, \dots, k$$
(2.13)

where $f_i(x)$ and $|f_i^{opt}|$ are respectively the i^{th} objective function value and its unique positive optimum value.

In situations where more than a single Pareto optimum solution is generated for consideration by the decision maker and consequent selection of an implementable one, it can be helpful, where possible, to develop a technique for assessing the solutions to aid the decision maker in selecting a preferred solution (Bagchi, 2014). In this work, a ranking scheme will be adopted to provide a single measure for the Pareto optimal objective function values in each solution. Specifically, a risk to expected return profile (RERP) measure given by:

$$RERP = \left(\sqrt{Z_A(x)} \Big/ E_A(x) \right) * 100\%$$
(2.14)

Which compares the standard deviation to the expected return for a Pareto optimal solution, will be used. The RERP will provide an objective basis for selecting a particular Pareto optimal solution for implementation, since lower values for the RERP are better than higher values when comparing both values of the objective functions.

3. Results and Discussions

In this section the investment problems of the two businesses, herein referred to as Investor A and Investor B, are modeled as bi-criteria optimization and applied to data obtained from the two for the purpose. The results from running the models are presented in the following succeeding sections and discussed.

3.1 Model for Investor A

Investor A runs a business in which he purchases a variety of goods and sells for profit. In the period studied, Investor A has four areas of investment with projected rates of returns over an. eight months period. Investor A has a fixed amount of Two Fifty Thousand Ghana Cedis (GHS 250,000) that he wants to invest in the four identifiable areas of investment over the known period of time. The desire is to determine how much to invest in the identifiable areas of investment in order to maximize return and minimize at the same time variability in the returns over the period. He has decided to invest up to the total capital available to him; however, he has determined the following restrictions: That the combined amounts invested in the first two investment areas (i.e. A1 and A2) and the last two investment areas (i.e. A3 and A4) respectively (see Table 4.1) should not exceed Ninety Thousand Ghana Cedis (GHS 90,000); at least Five Thousand Ghana Cedis (GHS 5,000) each should be invested in

investment areas A1 and A3, while at least Twenty Five Thousand Cedis (GHS 25,000) and Four Thousand Cedis (GHS 4,000) respectively must be invested in areas A2 and A4.
The detailed data associated with this problem is given in Table 3.1. The table shows the four investment areas and the projected rates of return over the eight (8) months period. In order to formulate the required objective functions based on the data in Table 3.1, Table 3.2 is constructed.

Table 3.1: Initial Data from Investor A

	Investment Areas & Rates of Return (%)			
Period	A1	A2	A3	A4
September 2020	12	20	10	15
October 2020	10	10	20	10
November 2020	15	10	10	10
December 2020	10	20	10	10
January 2021	10	5	10	15
February 2021	10	5	10	15
March 2021	10	10	5	5
April 2021	10	12	10	5

Table 3.2: Computing Necessary Parameters for Model of Investor A

K	r_{1k}	r_{2k}	r_{3k}	r_{4k}	r_{1k}^2	$r^2{}_{2k}$	r_{3k}^2	r_{4k}^2	$r_{1k}r_{2k}$	$r_{1k}r_{3k}$	$r_{1k}r_{4k}$	$r_{2k}r_{3k}$	$r_{2k}r_{4k}$	$r_{3k}r_{4k}$
1	12	20	10	15	144	400	100	225	240	120	180	200	300	150
2	10	10	20	10	100	100	400	100	100	200	100	200	100	200
3	15	10	10	10	225	100	100	100	150	150	150	100	100	100
4	10	20	10	10	100	400	100	100	200	100	100	200	200	100
5	10	5	10	15	100	25	100	225	50	100	150	50	75	150
6	10	5	5	15	100	25	25	225	50	50	150	25	75	75
7	10	10	10	5	100	100	100	25	100	100	50	100	50	50
8	10	12	12	5	100	144	144	25	120	120	50	144	60	60
Total	87	92	87	85	969	1294	1094	1025	1010	940	930	1019	960	885

The resulting model is:

$$Max\ E_A(x) = 87x_{A1} + 92x_{A2} + 87x_{A3} + 85x_{A4}$$

$$Min\ \ Z_A(x) = (x_{A1}\ x_{A2}\ x_{A3}\ x_{A4})\begin{bmatrix} 2.9 & 1.2 & -0.8 & 0.7 \\ 1.2 & 29.5 & 2.3 & -2.2 \\ -0.8 & 2.3 & 18.5 & -4.9 \\ 0.7 & -2.2 & -4.9 & 15.2 \end{bmatrix}\begin{pmatrix} x_{A1} \\ x_{A2} \\ x_{A3} \\ x_{A4} \end{pmatrix}$$

Subject to: $x_{A1} + x_{A2} + x_{A3} + x_{A4} \leq 250000$

$x_{A1} + x_{A2} \leq 90000;$
$x_{A3} + x_{A4} \leq 90000;$
$x_{A1} \geq 5000;$
$x_{A2} \geq 25000;$
$x_{A3} \geq 50000;$
$x_{A4} \geq 40000$

The ideal solutions or unique maximum and unique minimum of the objective functions obtained separately as:

$Max\ E_A(x) = 16{,}005{,}000$ and $Min\ \ Z_A(x)$ =26,070,000,000

The model with normalized objective functions is:

$$Max[\ w_i E_A(x)^{norm} - (1 - w_i)(Z_A(x)^{norm})]$$

Subject to: $x_{A1} + x_{A2} + x_{A3} + x_{A4} \leq 250000$
$x_{A1} + x_{A2} \leq 90000;$
$x_{A3} + x_{A4} \leq 90000;$
$x_{A1} \geq 5000;$
$x_{A2} \geq 25000;$
$x_{A3} \geq 50000;$
$x_{A4} \geq 40000$
$w_i + (1 - w_i) = 1, \quad w_i > 0\ \ \forall\ \ i$

where $E_A(x)^{norm}$ and $Z_A(x)^{norm}$ are the normalized objective functions and w_i is the i^{th} weight. By varying w_i from 0.1 to 0.9 in steps of 0.1, Pareto optimal solutions were generated for the model.

3.2 Solution of Model for Investor A

The resulting Pareto optimal solutions for the model of Investor A are displayed in Table 3.3. The table shows 20 weight pairs and the corresponding Pareto optimal solutions. The table indicates that weighting of the objective functions did not result in a wide variety of Pareto optimal solutions. The distinct Pareto optimal solutions displayed in Table 3.3 are extracted from it and presented in Table 3.4.

Table3.3: Pareto Optimal Solutions for Selected Weights for Model of Investor A

w_i	$(1-w_i)$	Pareto optimal variables Values				Optimum E(x) & Z(x) values	
		x_{A1}	x_{A2}	x_{A3}	x_{A4}	E(x)	Z(x)
0.1	0.9	5000	25000	50000	40000	104,450,000	55,140,000,000
0.2	0.8	5000	25000	50000	40000	104,450,000	55,140,000,000
0.3	0.7	5000	25000	50000	40000	104,450,000	55,140,000,000
0.4	0.6	5000	25000	50000	40000	104,450,000	55,140,000,000
0.5	0.5	5000	25000	50000	40000	104,450,000	55,140,000,000
0.6	0.4	5000	25000	50000	40000	104,450,000	55,140,000,000
0.7	0.3	5000	25000	50000	40000	104,450,000	55,140,000,000
0.8	0.2	5000	25000	50000	40000	104,450,000	55,140,000,000
0.9	0.1	5000	25000	50000	40000	104,450,000	55,140,000,000
0.99	0.01	61982	28018	50000	40000	15,720,090	76,720,000,000
0.98	0.02	65000	25000	50000	40000	15,720,090	76,720,000,000
0.97	0.03	65000	25000	50000	40000	15,720,090	76,720,000,000
0.96	0.04	65000	25000	50000	40000	15,720,090	76,720,000,000
0.95	0.05	65000	25000	50000	40000	15,705,000	77,090,000,000
0.94	0.06	65000	25000	50000	40000	15,705,000	77,090,000,000
0.93	0.07	65000	25000	50000	40000	15,705,000	77,090,000,000
0.92	0.08	65000	25000	50000	40000	15,705,000	77,090,000,000
0.91	0.09	65000	25000	50000	40000	15,705,000	77,090,000,000
0.90	0.1	65000	25000	50000	40000	15,705,000	77,090,000,000
0.89	0.11	65000	25000	50000	40000	15,705,000	77,090,000,000

Table 3.4: Varied Sets of Pareto Optimal Solutions for Investor A

Variables	Optimum Values		
x_{A1}	5,000	65,000	61,982
x_{A2}	25,000	25,000	28,018
x_{A3}	50,000	50,000	50,000
x_{A4}	40,000	40,000	40,000
$E_A(x)$	104,450,000	15,705,000	15,720,090
$Z_A(x)$	55,140,000,000	77,090,000,000	76,620,000,000
$SD = \sqrt{Z_A(x)}$	234,819	277,651	276,803
RERP	0.225%	1.768%	1.761%

3.3 Model for Investor B

Unlike Investor A, Investor B invests in three main areas which are distinct from those of Investor A with their projected rates of return covering a 12 month period as given in Table 3.5. For the period under study, Investor B had up to 220,000 Cedis Capital to invest in the three areas of investment B1, B2, and B3. He had determined that it is prudent not to invest more than 90,000 in investment areas B1 and B2 combined. Similarly, he had determined that the combined investments in areas B1 and B3 should not exceed 100,000 Cedis, while the combined investments of areas B2 and B3 should not exceed 200,000. His desire was to maximize his return over the period of the investments and minimize the variability of his return over the period.

Since Investor B's problem is similar to that of Investor A's, the same procedures applied to model, solve and analyse Investor A's problem are applied to investor B's problem. Furthermore, that would facilitate the undertaking of further investigations, later in a second paper, about the sensitivity of the models and possible benefit of a joint investment by the two businesses.

Table 3.5: Initial Data from Investor B

Period	Areas and Rates of Returns (%)		
	B1	B2	B3
January	20	15	20
February	20	15	10
March	15	15	15
April	20	25	10
May	20	10	10
June	10	18	20
July	15	20	30
August	15	15	20
September	10	15	15
October	5	20	10
November	10	15	20
December	20	30	10

Table 3.6: Computed Necessary Parameters for Model of Investor B

k	r_{1k}	r_{2k}	r_{3k}	r_{1k}^2	r_{2k}^2	r_{3k}^2	$r_{1k}r_{2k}$	$r_{1k}r_{3k}$	$r_{2k}r_{3k}$
1	20	15	20	400	225	400	300	400	300
2	20	15	10	400	225	100	300	200	150
3	15	15	15	225	225	225	300	225	225
4	20	25	10	400	625	100	500	200	250
5	20	10	10	400	100	100	200	200	100
6	10	18	20	100	324	400	180	200	360
7	15	20	30	225	400	900	300	450	600
8	15	15	20	225	225	400	225	300	300
9	10	15	15	100	225	225	150	150	250
10	5	20	10	25	400	100	100	50	200
11	10	15	20	100	225	400	150	200	300
12	20	20	30	400	400	900	400	600	600
Total	180	203	210	3000	3599	3890	3105	3175	3610

The expected value function $E_B(x)$ and the variance function $Z_B(x)$ which denote the objective functions of Investor B, where $x = (x_{B1}, x_{B2}, x_{B3})^T$, are constructed by first computing the values in Table 3.6. As was done for Investor A, the Weighted Sum model for investor B using normalized objective functions is presented below:

$$Max[w_i E_B(x)^{norm} - (1 - w_i) Z_B(x)^{norm}]$$

Subject to:
$$x_{B1} + x_{B2} + x_{B3} \leq 220000$$
$$x_{B1} + x_{B2} \leq 90000$$
$$x_{B1} + x_{B3} \leq 100000$$
$$x_{B2} + x_{B3} \leq 200000$$
$$30000 \leq x_{B1} \leq 50000$$
$$25000 \leq x_{B2} \leq 90000$$
$$20000 \leq xB_3 \leq 200000$$
$$w_i > 0, \ w_i + (1 - w_i) = 1 \ \forall i$$

3.4 Solution of Model of Investor B

Similar to the solution outcome for the model of Investor A, the current model produced mostly the same Pareto optimal solutions with variations of weight sets. The two distinct results obtained for weight sets {0.05, 0.95} and {0.9, 0.1} in all the variations are presented in Table 3.7.

Table 3.7: Pareto Optimal solutions for Investor B

Variables	Optimum values	
$x_{B1}(x)$	30,000	32,059
$x_{B2}(x)$	25,000	57,941
$x_{B3}(x)$	20000	61,090
$E_B(x)$	14,675,000	30,361,543
$Z_B(x)$	55,832,500,000	294,177,317,900
$SD = \sqrt{Z_B(x)}$	236,289	542,381
RERP	0.016%	0.017%

3.5 Discussions

Table 3.4 presents three sets of Pareto optimal solutions. The first indicates that Investor A could expect to make about 104.5M Ghana Cedis return with a standard deviation of about 234.8K Ghana Cedis on his investments, if he invests 5K, 25K, 50K, and 40K Ghana Cedis respectively in investment areas A1, A2, A3 and A4. The second set indicates that the investor could make 15.7M Ghana Cedis with standard deviation of 277.8K Ghana Cedis, if he invests 65K, 25K, 50K, and 40K Ghana Cedis respectively in the areas A1 to A4. Similar interpretation follows for the third set. The corresponding standard deviations which measure the risk levels for the investments were about 234.8K, 277.6K, and 276.8K Ghana Cedis respectively. They indicate that the expected returns for each of the three sets of investments could vary upwards or downwards by up to those margins. The RERP values indicate that the first set of solutions yields a least value of 0.2%. Therefore, Investor A could select that for implementation.

Similar interpretation as given for the solutions for Investor A applies to the results displayed in Table 3.5 for Investor B. Therefore, for instance (from the first set of solutions in Table 3.5), Investor B could expect to make 14.6M Ghana Cedis if he invests 30K, 25K and 20K Ghana Cedis respectively in B1, B2, and B3. However, his return can vary upward or downward by margins of up to 236.2K Ghana Cedis. In terms of risk to return profile given by RERP values, Investor be may select the solution with RERP value of 0.016% for implementation.

It is clear that the weighted sum scalarization solution method did not have any significant impact on the model as far as finding diverse Pareto optimal solutions is concerned and this cannot be attributed to a non-convex Pareto front (the optimization models were convex), nor to many objective functions (there were only two), Therefore, that not many diverse solutions were obtained could be because the Pareto fronts were weakly convex and thus solutions hardly changed from one point on their surfaces to another. This is worth investigating in a future research using other scalar methods.
The solutions themselves indicated that the two investors did not have to invest their entire capital in the various investment areas. This could be dictated by their operational restrictions which were the constraints of the models. Nevertheless the results showed that the investors could still make sizable returns and therefore profits over their investment periods, taking into account however the risks to expected return profiles.

4. Conclusions

This work sought to investigate, from a multi-objective optimization stand-point the outcomes for two separate investment problems of two businesses in Ghana. The aim was to optimize separately but simultaneously their risks and returns on their investments based on real data collected separately from them. A bi-objective nonlinear constrained convex optimization model was thus formulated for the two. The results have indicated that maximal returns and minimal risks are possible for the two businesses even though only two or three possibilities were available for them to consider.

The two or three possible solution sets are suspected to be the result of weakly convex Pareto fronts that did not allow for large diversity of solutions. This, however, can be invested further in a future work. Other solution methods can also be used, such as the constraint method. As an immediate follow-up to this study, to be reported as Part 2 to this paper, investigation of the sensitivity of the models to parameter variations would be explored. Furthermore, the prospects for a joint investment of the two businesses would be investigated.

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